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## Introduction to Core Maths

# INDUCTION BOOKLET

## Introduction

Thank you for choosing to study Core Maths in the sixth form.

The Mathematics Department is committed to ensuring that you make good progress throughout your Core Maths course. In order that you make the best possible start to the course, we have prepared this booklet. It is vital that you spend time working through the questions in this booklet over the summer. You need to have a good knowledge of these topics before you commence your course in September. You should have met all the topics before at GCSE.

Work through what you need to from each chapter, prioritising topics you are less comfortable with. Make sure that you understand the examples, then tackle the exercise to ensure you understand the topic thoroughly. The answers are at the back of the booklet. You will need to be very organised so keep your work in a folder & mark any queries to ask at the beginning of term.

Use this introduction to give you a good start to your Year 12 work that will help you to enjoy, and benefit from, the course. The more effort you put in, right from the start, the better you will do.

Sources for further help are indicated throughout the booklet. All the topics can be found on resources such as Corbett Maths, Hegarty Maths or Dr Frost Maths.

<https://hegartymaths.com/>

<https://www.drfrostmaths.com/>

<https://corbettmaths.com/>

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# 1. Data tables, averages and spread

## 1.1 Averages

For support with this section, have a look at Hegarty clips 404 – 410.

### Mean

To find the mean, add all your numbers together and divide by how many numbers you have.

#### Example

The mean of 6, 3, 8 and 7 is:

$$\frac{6 + 3 + 8 + 7}{4} = \frac{24}{4} = \mathbf{6}$$

### Mode

The mode is the number that occurs most often.

#### Example

The mode of 3, 6, 9, 3, 1, 7, 8, 4 is: **3**

### Median

When the list in order, the median is the number that is in the middle of the list.

#### Example

The median of 6, 3, 5, 8 and 7 is:

~~3, 5, 6, 7, 8~~

The median of 1, 8, 5 and 10 is:

~~1, 5, 8, 10~~

The median is between 5 and 8, so  $\frac{5+8}{2} = \mathbf{6.5}$

### Range

The range is the value you get when you take the smallest number from the biggest number.

The range is NOT an average it is a measure of spread

If the range is small, there is not much variation in the numbers. This means the data is consistent. The smaller the range, the more consistent the data.

#### Example

The range of 6, 3, 8 and 7 is:  $8 - 3 = \mathbf{5}$

## Data tables, averages and spread: Practice sheet 1

1. Himanji made a list of her homework marks for the last ten weeks:

14    15    15    15    14    13    11    12    14    15

- (a) Write down the mode of her homework marks  
(b) Work out her mean homework mark

2. Here is a set of numbers:

8    7    3    5    2

Add a number to ensure you get a median of 6

3. The test marks for 8 girls and 6 boys are shown:

Girls:	15	13	18	12	17	13	13	18
Boys:	12	15	19	13	15	7		

- (a) Work out the median mark of the boys.  
(b) Work out the range of the girls.  
(c) Are the girls' marks more consistent than the boys' marks? Explain your answer.

4. A football team played 7 games. The number of goals scored in each game was:

4    2    1    5    0    1    4

Work out the median number of goals.

In the next game the football team scores 4 goals.

Find the median number of goals scored in the 8 games.

5. Ursula is training to walk up the Machu Picchu trail in Peru. This is 43km long. In four of the five weeks leading up to the walk she has walked

42km    48km    41km    38km

Ursula thinks her mean for the four weeks is more than this.

Is she right?

She wants to increase her mean so that it is 45km. How far should she walk on the fifth week?

Answers: Page 57

## 1.2 Averages from frequency tables

For support with this section, have a look at Hegarty clips 415 – 417.

### Mean

length	freq	Show data
38	3	38 + 38 + 38 (3 × 38)
42	5	+ 42 + 42 + 42 + 42 + 42 (5 × 42)
43	4	+ 43 + 43 + 43 + 43 (4 × 43)

1. Add up all the values

Find mean

$$3 \times 38 + 5 \times 42 + 4 \times 43$$

$$= 114 + 210 + 172 = 496$$

2. Divide by the number of values  $3 + 5 + 4 = 12$

$$\text{mean} = \frac{496}{12} = 41.33$$

### Mode

The mode is the value which occurs the most. In a table, this is the value that has the highest frequency.

#### Example

Number of flowers	Freq
7	3
8	2
9	4
10	7
11	6

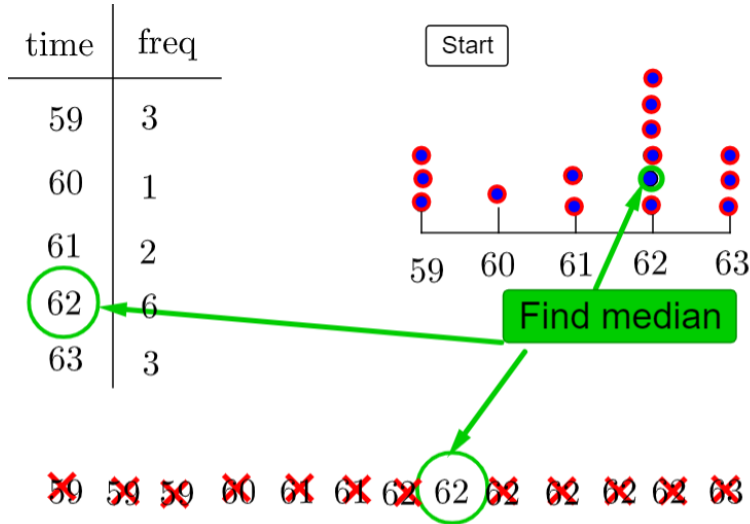
The highest frequency in this table is 7, which corresponds to **10 flowers**.

Therefore, the mode is **10**.

## Median

As above, we need to find the middle value from the data. We can do this by writing out the list of data from the table, or by find the value in the  $\frac{n+1}{2}$  position, where n is the total frequency.

### Example



Or:

Total frequency =  
 $3+1+2+6+3 = 15$

$$\frac{n+1}{2} = \frac{15+1}{2}$$

$$= 8^{\text{th}} \text{ position}$$

So median = **62**

### 1.3 Averages from grouped tables

For support with this section, have a look at Hegarty clips 415, 416 & 418.

#### Mean

To find the mean from a grouped frequency table:

1. Calculate the midpoints of the groups
2. Calculate the midpoint x frequency for each row
3. Find the total frequency and total midpoint x frequency
4. Estimated Mean = total midpoint x frequency  $\div$  total frequency

#### Example

Number( $n$ )	Midpoint		Freq
$40 < n \leq 45$	42.5	$\times$	5
$45 < n \leq 50$	47.5	$\times$	7
$50 < n \leq 55$	52.5	$\times$	3
$55 < n \leq 60$	57.5	$\times$	5

$$\frac{42.5 \times 5 + 47.5 \times 7 + 52.5 \times 3 + 57.5 \times 5}{5 + 7 + 3 + 5}$$

$$= \frac{212.5 + 332.5 + 157.5 + 287.5}{20}$$

$$= \frac{990}{20} = 49.5$$

#### Mode

When the data are grouped, the modal group is the group with the highest frequency.

#### Example

Time	Freq	
$0 < t \leq 4$	14	The highest frequency is 22.
$4 < t \leq 8$	22	Therefore, the modal group is $4 < t \leq 8$ .
$8 < t \leq 12$	21	
$12 < t \leq 16$	8	
$16 < t < 20$	5	



## Median

As with the previous table, we can estimate the median by finding the value in the  $\frac{n+1}{2}$  position, where n is the total frequency.

### Example

Time	Freq	
$0 < t \leq 4$	14	Total frequency = $14 + 22 + 21 + 8 + 5 = 70$
$4 < t \leq 8$	22	$\frac{n+1}{2} = \frac{70+1}{2} = \frac{71}{2} = 35.5$
$8 < t \leq 12$	21	
$12 < t \leq 16$	8	14 so far in $0 < t \leq 4$ group. If we count on, there's 36 between 0 and 8.
$16 < t < 20$	5	Median group: $4 < t \leq 8$

## Data tables, averages and spread: Practice sheet 2

1. Jane asked 20 people how many chocolate bars they ate last week. The results are shown in the table:

Number of chocolate bars	Number of people
0	4
1	3
2	6
3	1
4	0
5	4
6	1
7	1

- (a) Write down the modal number of chocolate bars eaten.
- (b) Work out the median number of chocolate bars eaten.
- (c) Calculate the mean number of chocolate bars eaten.

2. The heights of students in a class are measured. The results are shown in the table:

Height interval ( $h$ cm)	Number of students
$160 \leq h < 165$	4
$165 \leq h < 170$	7
$170 \leq h < 175$	5
$175 \leq h < 180$	3
$180 \leq h < 185$	1

- Write down the group in which the median lies.
- Work out an estimate of the mean height of the students.
- Why is this an estimate?
- Margret says that she thinks the class has both boys and girls in it. Give a reason why the heights in the class are consistent with Margret's opinion.

3. A group of nine students marked their shoe sizes on the dot plot below.



- Find the mode, median and mean shoe size in the group.
- What do you notice?
- Two more students add their shoe sizes. They are size 1 and size 9. Mark these on the dot plot.
- What are; the mode, median and mean now?
- By considering the dot plot, explain why this has happened.

4. Each year, more than 150 countries are surveyed for the World Happiness Report.

The main question asked is: "Please imagine a ladder, with steps numbered from 0 at the bottom to 10 at the top. The top of the ladder represents the best possible life for you and the bottom of the ladder represents the worst possible life for you. On which step of the ladder would you say you personally feel you stand at this time?"

A psychology teacher wants to use this question to compare the happiness levels in their two Year 12 classes.

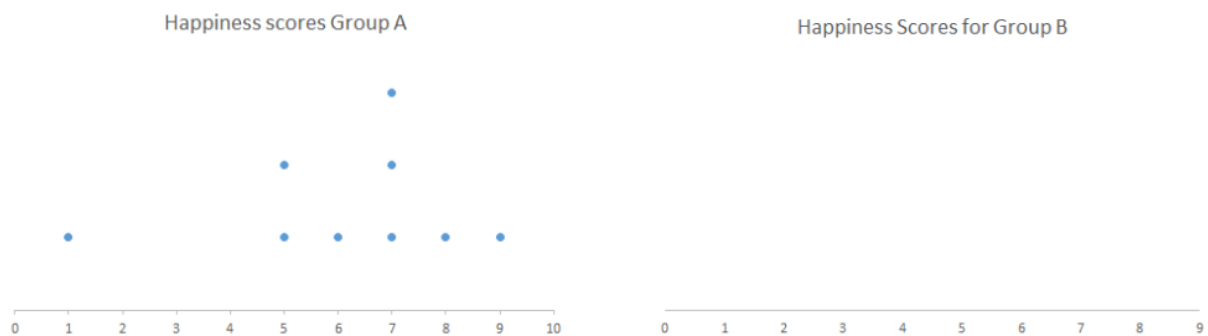
The results are in the table below.

Group A	7	6	5	7	8	1	5	7	9
Group B	8	6	6	5	6	6	7	7	

a) Before you do any calculations, which group do you think is happiest and why?

The dot plot below shows the happiness scores for Group A.

b) Complete the dot plot for Group B.



c) Calculate the mean, median, mode and range for Group A and Group B.

d) Based on your results and by comparing the dot plots, would you change your answer to part (a)? Please explain why.

Answers: Page 59

## 2. Accuracy and estimation

### 2.1 Accuracy

For support with this section, have a look at Hegarty clips 17, 56, 130.

The accuracy used depends on the size of the figure being quoted.

We need the answer to be precise enough, but being too accurate is unnecessary and sometimes unrealistic. For example, if the population of a town is 34,567, you probably wouldn't say the entire number when describing it. You are more likely to say "around 35,000" or "around 34600".

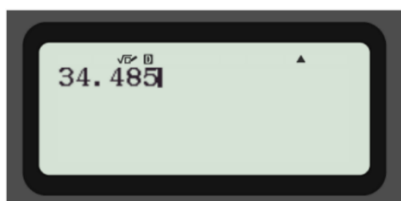
Accuracy is also often limited by the measuring instrument used. For example, a stopwatch can usually only measure to 2 d.p, and a school ruler can usually only measure to the nearest mm.

#### Decimal places

This system only applies to numbers with a decimal point.

When rounding to 1 d.p, we are rounding to the nearest 10th, and our answer will have 1 digit after the decimal point. When rounding to 2 d.p, we are rounding to the nearest 100th, and our answer will have two numbers after the decimal point.

#### Example



This answer when rounded to 1 d.p. is 34.5

#### Significant figures

Significant figures provide us with a useful system for rounding numbers of any size, including both whole numbers and decimals. They are widely used in the real world, particularly in science.

When rounding to 1 s.f, we look at the first digit (ignoring any zeros at the beginning), which tells us the accuracy we are rounding to.

#### Example

Round 34.485 to 1 s.f.

The first digit of 34.485 is 3. This is in the 10s column, so we are rounding to the nearest 10.

34.485 is closer to 30 than 40, so our answer rounded to one significant figure is **30**.

## Core Maths Skills - Accuracy worksheet

### Accuracy:

1. When taking a measurement, the accuracy is often limited by the measuring device. Complete the sentence using one of the four options:

A ruler measures to the nearest .....

- A. m
- B. cm
- C. mm
- D. 0.1 mm

2. Sometimes we round numbers as it isn't necessary to know the exact value. There are 43,245 students in a university. This is quoted as "around 43,000 students". What accuracy has this been rounded to? Select one option.

- A. Nearest 10,000
- B. Nearest 1,000
- C. Nearest 100
- D. Nearest 10



3. Round the following numbers to 1 d.p:
- a) 2.576
  - b) 19.541
  - c) 4.99
4. Round the following numbers to 2 d.p:
- a) 6.371
  - b) 14.7979
  - c) 132.4555
5. Round the following numbers to 1 s.f:
- a) 345
  - b) 3.653
  - c) 0.00201
6. Round the following numbers to 2 s.f:
- a) 24,540
  - b) 0.699
  - c) 19.598

Answers: Page 64

## 2.2 Estimation

For support with this section, have a look at Hegarty clips 131.

We can use estimation answer questions by doing a similar calculation, but easier, to give us an idea of the answer.

### Example

In an exam, a student wrote 1482 words in 2 hrs 50 mins. Estimate how many words they wrote each hour.

1482 words is close to 1500 words

2 hrs 50 mins is close to 3 hrs

$1500 \div 3 = \mathbf{500 \text{ words}}$

You are used to estimating calculations from GCSE but you now have to be able to estimate real life problems.

When estimating, we usually round the numbers to make calculations easier – choose the most suitable in each case.

## Core Maths Skills - Estimation worksheet

### Estimation:

1. Estimate the sizes of the objects A – D. Match them up with the most suitable measurement I – IV.

A. The length of a football pitch	I. 2m
B. The height of a tree	II. 10m
C. The length of a large swimming pool	III. 100m
D. The height of a standard door	IV. 50m

2. We often approximate numbers in a calculation in order to make an estimate. Estimate the following calculations, choosing the most appropriate final answer.

a)  $\frac{2.4 \times 139}{6.88}$

- I. 0.4
- II. 4
- III. 40
- IV. 400

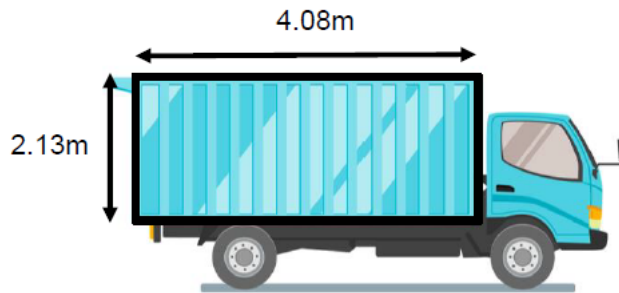
- b) A builder lays 114 bricks in 4 hrs 8 minutes. Estimate how many bricks they lay per hour.

- I. 25
- II. 3
- III. 20
- IV. 250

c) An office worker drives 38.7 miles **to and from** work each day. Estimate how far they drive in a working week (they only work weekdays).

- I. 200 miles
- II. 400 miles
- III. 150 miles
- IV. 300 miles

3. The truck below is 1.8m wide. Estimate the volume (in  $\text{m}^3$ ) of the container on the back of the truck.



Answers: Page 64

### 3. Interpreting graphs and charts (statistics for life)

#### 3.1 Revising statistical diagrams

For support with this section, have a look at Hegarty clips 393, 425, 427-429.

**Discrete** data can only take exact values. The unit of measurement cannot be split up. For example the number of students in each class.

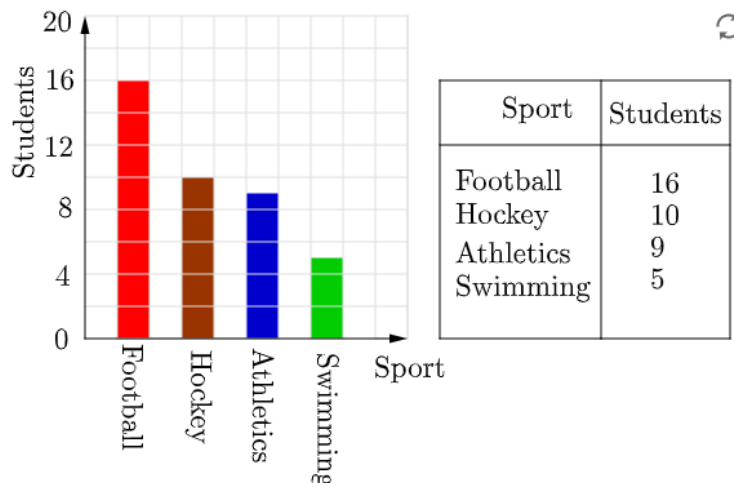
**Continuous** data can take any value. This means every point has a value. For example height is continuous data as heights are not just 175cm or 176cm, but every value in between too.

#### Bar charts

A bar chart is a way of representing data using rectangular bars.

##### Example

In this bar chart the number of students who do different sports are shown.

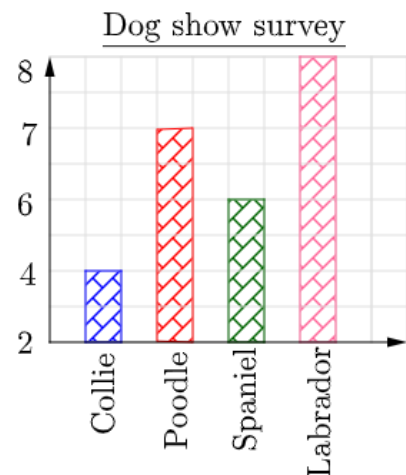


Sometimes bar charts can be misleading.

##### Example

This bar chart represents the number of dogs at a dog show. There are three mistakes in this bar chart.

- The numbers don't go up evenly.
- The vertical axis doesn't start at 0.
- There are no labels on the axes.





### Pie charts

In every pie chart the angles all add up to  $360^\circ$ . Pie charts are used to show different proportions of the whole. You often need to work out how many degrees will represent one person.

#### Example

Total number of people = **45**

$$\text{Angle for one person} = \frac{360}{45} = 8^\circ$$

#### Example

There are a number of students in a course. Each must study a science subject. The pie chart represents the number of students who chose each of the sciences.

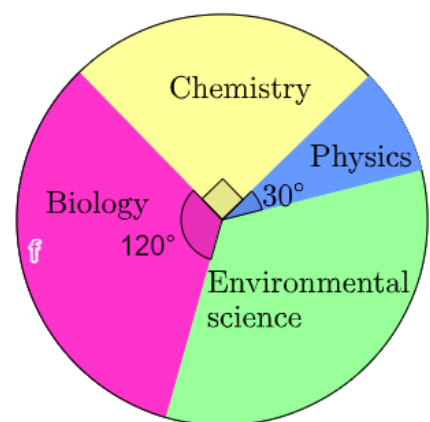
15 chose physics.

How many chose environmental science?

From physics: Angle for 1 person =  $30 \div 15 = 2^\circ$

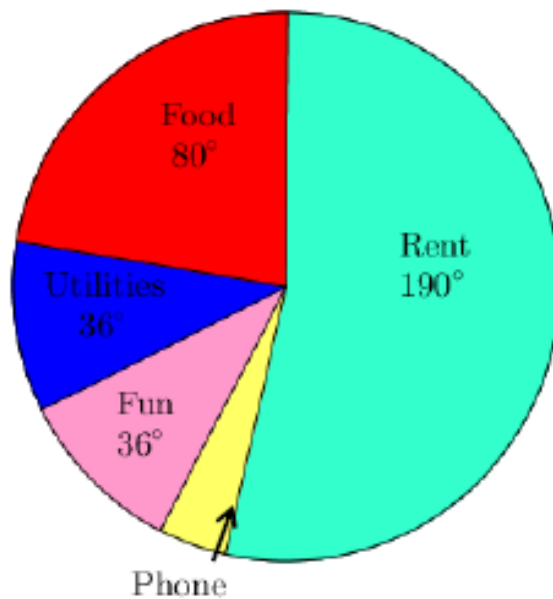
Environmental science angle =  $360 - 120 - 90 - 30 = 120^\circ$

$120 \div 2 = \mathbf{60 \text{ people}}$



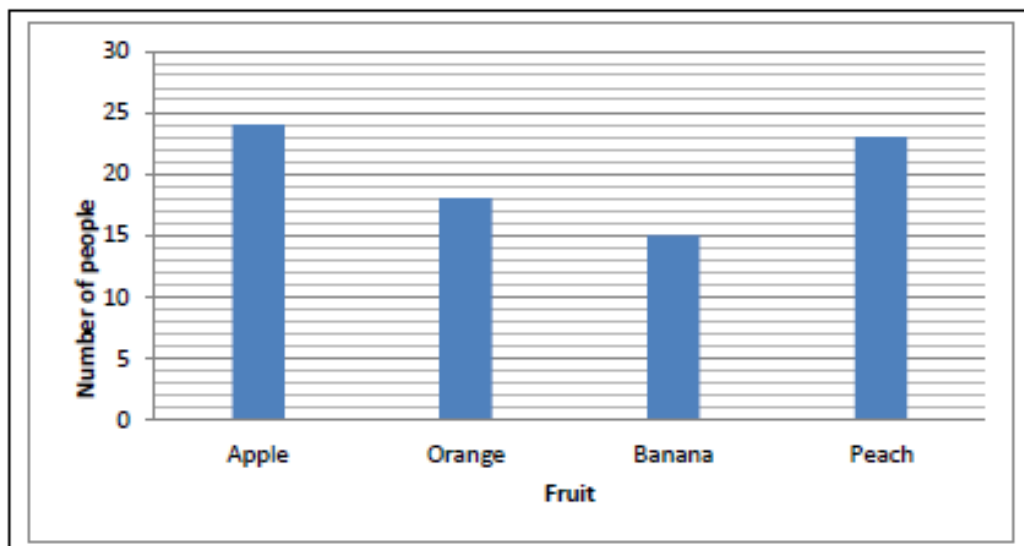
## Practice sheet: Statistical diagrams

1. This pie chart shows the monthly budget for a student, Adi.



- What is the angle of the sector for "Phone"?
- Adi spends £380 on rent each month. How much does he spend on his phone?

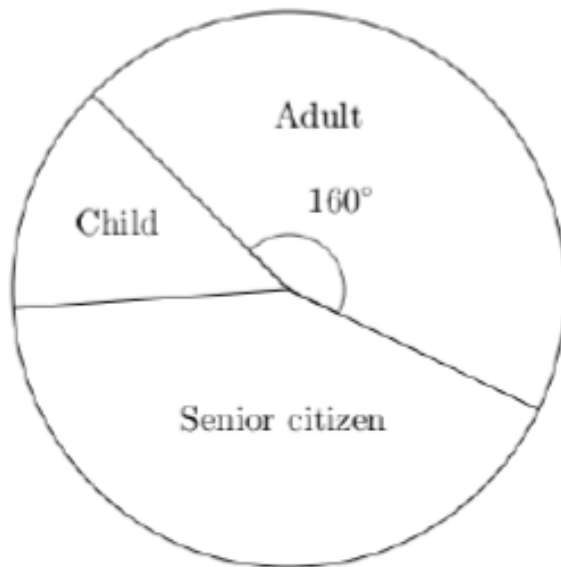
2. A survey was conducted to determine the favourite fruit of 80 people. The bar chart is shown:



- How many more people preferred apples to bananas?
- Draw and label a pie chart to represent this data.

(No question 3 or 4 for this section)

5. The pie chart shows the sale of 720 train tickets:



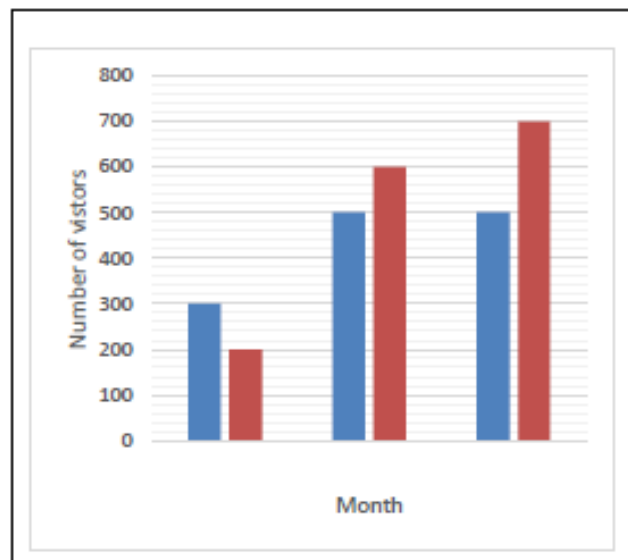
There were three times as many senior citizen tickets sold as child tickets.

How many child tickets were sold?

6. The table shows the number of male and female visitors at a garden for three months:

The same information is also shown on the bar chart:

Month	Male	Female
May	300	200
June	500	600
July	500	700



There are three mistakes on the bar chart. What are they?

Answers: Page 65

### 3.2 Scatter diagrams

For support with this section, have a look at Hegarty clips 453, 454.

**Bivariate** data is data which is paired. Examples include:

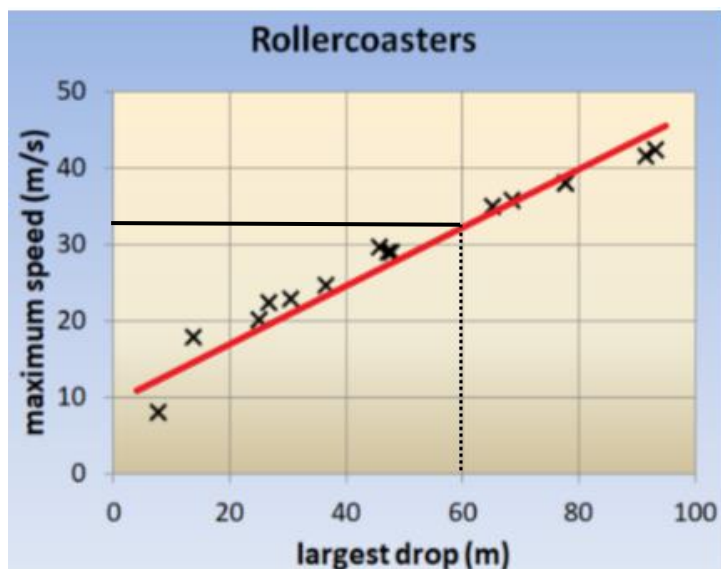
- (population, land area) for countries
- (power, weight) for cyclists
- (mass, price) for breakfast cereals
- (CD sales, downloads) for albums
- 

A scatter diagram is used for **numerical** bivariate data

Sometimes the data shows a pattern which can be approximated by a **line of best fit**. In the diagram below, the line rises from bottom left to top right. This is referred to as a **positive correlation**. The line can be used to estimate values.

#### Example

A roller coaster has a largest drop of 60m. Estimate its max speed.



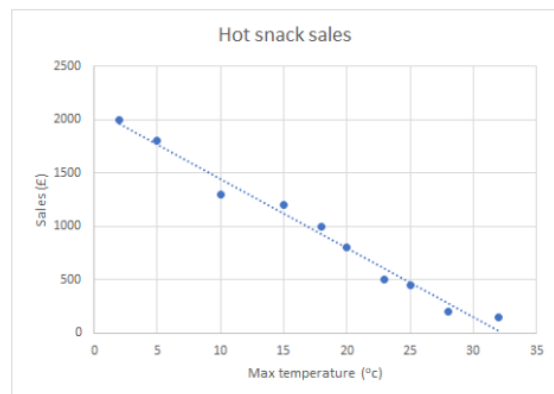
Find 60m on the horizontal axis.

Go up from 60m to the line of best fit.

Go across to the other axis and read off the value: about **32m/s**

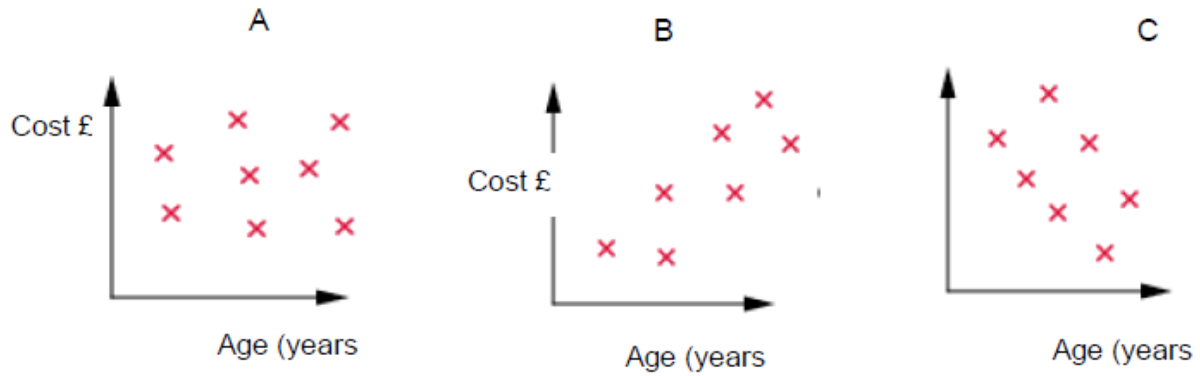
#### Example

The trendline for this set of data slopes from top right to bottom left. This is called a **negative correlation**. This diagram seems to show that as the temperature increases, the amount earned from hot snack sales decrease.



## Practice sheet: Introduction to scatter diagrams

1) Here are three scatter graphs showing the age of second hand cars and their cost:



Allan says "the older a car, the more it costs"

Jane says "the older a car, the less it costs"

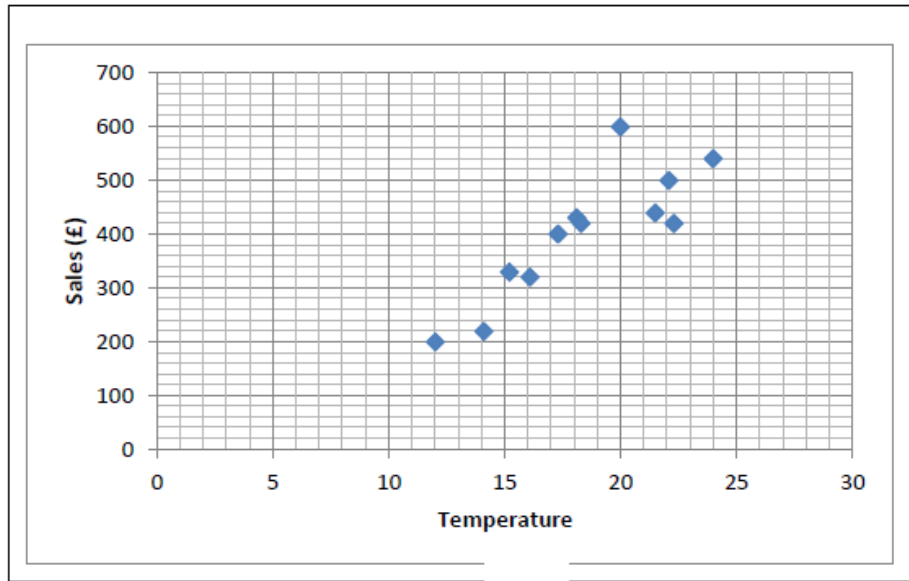
Ranjit says "there is no relationship between the age of a car and its cost"

Match a scatter graph to each statement.

2) Here is a scatter diagram with four points shown. Add 8 points anywhere on the diagram to make a negative correlation



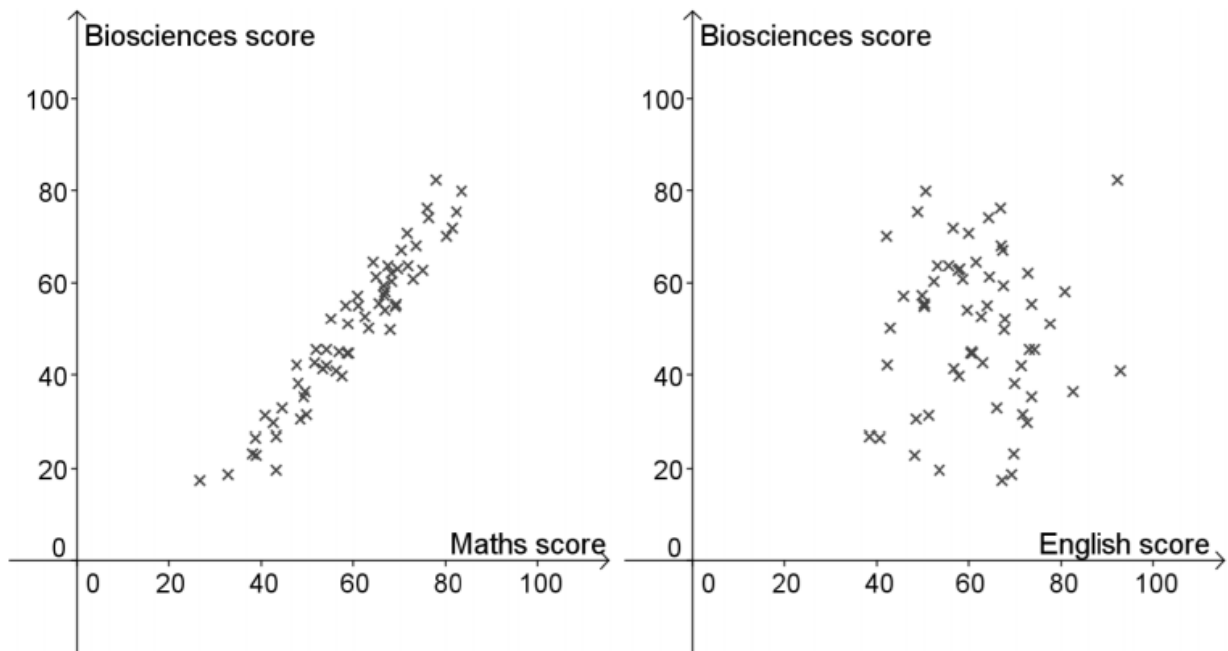
- 3) An ice cream salesman keeps track of the amount of money taken in ice cream sales against the temperature at midday. The graph below shows this:



- Describe the relationship between the amount of money taken and the temperature at midday.
- Draw in the line of best fit.
- Estimate the amount of money that will be taken if the temperature at midday is  $20^{\circ}\text{C}$ .
- One of the values is an outlier. Give a possible reason for this.

4) A biosciences lecturer believes that success in the subject depends on being good at both maths and English. The lecturer wants to investigate whether test scores might show this. Sixty students sit a test in each of maths, English and biosciences. The results are shown in the two scatter diagrams.

**Note** - These data are not real. The scenario is designed to help you think about scatter diagrams.



Label the crosses representing the scores of the people described below. Each person should be labelled on both scatter diagrams.

- a) Ahmed, who got the highest score on the biosciences test
- b) Bethan, who got the lowest score in the maths test
- c) Callista, who got around 40 in all three tests

- 5) For each statement, decide how true it is and give a reason.
- (a) 5% of the students scored 20 or less on the biosciences test.
  
  - (b) One student scored over 80 on all three tests.
  
  - (c) There is strong positive correlation between maths scores and biosciences scores.
  
  - (d) If you know that Gemma scored lower than Hui on maths, then it follows that Gemma scored lower than Hui on biosciences.
  
  - (e) There is a closer correlation between maths and biosciences scores than between English and biosciences scores.
  
  - (f) Maths scores are a better predictor of success at biosciences than English scores.
  
  - (g) Success at maths is a bigger influence on success in biosciences than English.
  
  - (i) The lecturer should give more weight to maths grades than English grades when choosing students for the biosciences course.

Answers: Page 67



## 4. Standard form

For support with this section, have a look at Hegarty clips **128**, 122-127.

**Standard form** is when a number is written in the format:  $a \times 10^n$

The first part, a, needs to be a number between 1 and 10.

The second part  $\times 10^n$  indicates the column (place value) of the left most digit of the number.

### Example

$$2,350,000 = 2.35 \times 10^6$$

Standard form also covers small numbers.

Unit	Tenths	Hundredth	Thousandth
1	0.1	0.01	0.001
$10^0$	$10^{-1}$	$10^{-2}$	$10^{-3}$

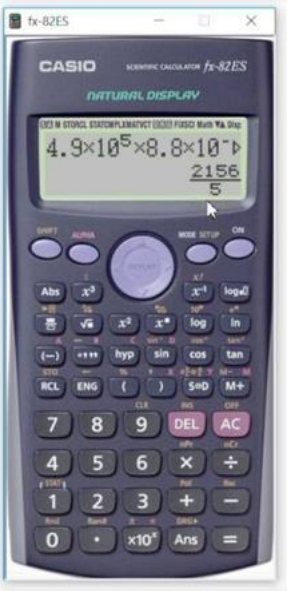
### Example

$$0.053 \text{ is written in standard form as } 5.3 \times 10^{-2}$$

You can use your calculator to work out standard form calculations.

### Example

a)  $4.9 \times 10^5 \times 8.8 \times 10^{-4}$



The image shows a Casio fx-82ES scientific calculator with a natural display. The screen shows the calculation  $4.9 \times 10^5 \times 8.8 \times 10^{-4}$  and the result  $2156.8$ . The calculator is a dark blue color with a numeric keypad and various function keys.

## Standard index form - Using a calculator

Use your calculator to work out the answers to the questions below.

1.  $(5.2 \times 10^4) \div (2.5 \times 10^3)$
2.  $(5.2 \times 10^4) \times (2.5 \times 10^3)$
3.  $(3.8 \times 10^{-4}) \div (2.6 \times 10^{-2})$
4.  $(8.4 \times 10^{-4}) \times (7.6 \times 10^{-3})$
  
5. On average an adult has 20,000,000,000,000 red blood cells.  
The mass of a typical red blood cell is 0.0000000001g.
  - a. Write both numbers in standard form.
  
  - b. Work out the mass of red blood cells in an average adult.

6. Worldwide, in 2018, there were about 37.8 million flight departures.
  - a. Write this number in standard form

26.3% of these flight departures were European.

- b. Work out the number of European flights in 2018

7. The approximate radius of the Earth according to google is 6371 km.

Assume the Earth is approximately a sphere.

- a. Use the formula to Surface area of a sphere =  $4\pi r^2$  (where r stands for the radius), to work out the surface area of the Earth in  $\text{km}^2$ .

- b. A website quotes the surface area of the area of the Earth as  $5.101 \times 10^7 \text{ km}^2$  is this correct?

Answers: Page 71

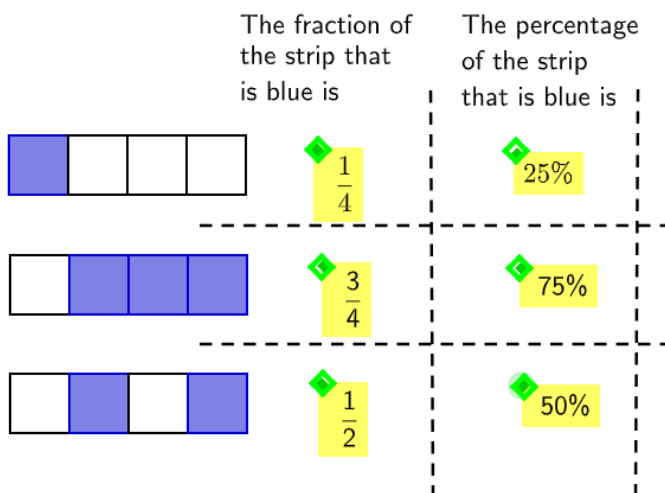
## 5. Percentages

### 5.1 Fractions, decimals and percentages

For support with this section, have a look at Hegarty clips 73-76, 82-83.

There are several different ways to express proportions. Fractions, decimals and percentages are all closely related.

In this example we use fractions and percentages.



Fraction to decimal: Doing (numerator  $\div$  denominator) on a calculator will find the decimal equivalent of a fraction. If the answer on the calculator display is shown as a fraction, press the  $S \Leftrightarrow D$  button to convert it to a decimal.

Decimal to percentage: Multiply a decimal by 100 to convert to a decimal.

Percentage to decimal: Divide a percentage by 100 to convert to a decimal.

Example

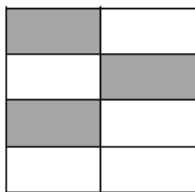
$$\frac{40}{100} = \text{Decimal } 0.4 = \text{Percentage } 40\%$$

Type  $40 \div 100$  on your calculator to get **0.4**.  
 $0.4 \times 100 = \mathbf{40\%}$

## Percentages, fractions and decimals

### Practice sheet 1

1. Complete the three statements about the diagram below.



The fraction of the shape that is shaded is .....

The percentage of the shape that is shaded is .....

2. Complete the statements below

(a) ..... out of 10 = 60%

(b) ..... out of 5 = 20%

3. Here are four numbers.

0.56     $\frac{67}{100}$     35%     $\frac{5}{8}$

Place the numbers in size order starting with the smallest.

4. Use this information  $\frac{12}{25} = 0.48 = 48\%$ ,  $\frac{13}{25} = 0.52 = 52\%$  and  $\frac{14}{25} = 0.56 = 56\%$  to help you complete the statements below:

$$\frac{16}{25} = 0.64 = \dots\dots\dots\%$$

$$\frac{17}{25} = \dots\dots\dots = \dots\dots\dots\%$$

$$\frac{\quad}{25} = 0.24 = \dots\dots\dots\%$$

$$\frac{\quad}{25} = 0.76 = \dots\dots\dots\%$$

$$\frac{\quad}{25} = 0.36 = \dots\dots\dots\%$$

$$\frac{\quad}{25} = 0.44 = \dots\dots\dots\%$$

$$\frac{\quad}{25} = \dots\dots\dots = 40\%$$

$$\frac{\quad}{25} = \dots\dots\dots = 60\%$$

5. Below is the mixture for a fruit drink. Complete the table by filling in the missing information.

Juice	Amount in the fruit drink	Percentage of the fruit drink
Grapefruit	$\frac{3}{8}$ litre	?
Mango	$\frac{1}{8}$ litre	?
Orange	$\frac{1}{2}$ litre	50%

6. A blackcurrant drink is made by mixing 35ml of concentrate with 170ml of water.

(a) What percentage of the drink is water? Give your answer to the nearest 1%

(b) Alisa mixes 20ml of concentrate with 80ml of water. Does this have a higher concentration of blackcurrant juice than the standard mix of 35ml concentrate with 170ml of water?

7.(a) Julia scored  $\frac{58}{65}$  in a French test. Abdullah scored  $\frac{78}{85}$  in a Spanish test.

(i) By comparing the scores as percentages decide whose performance was better.

(ii) Give a reason why using the percentage score may not be a good way to compare Julia and Abdullah's performance.

(b)The results from three of Jason's course modules are shown below.

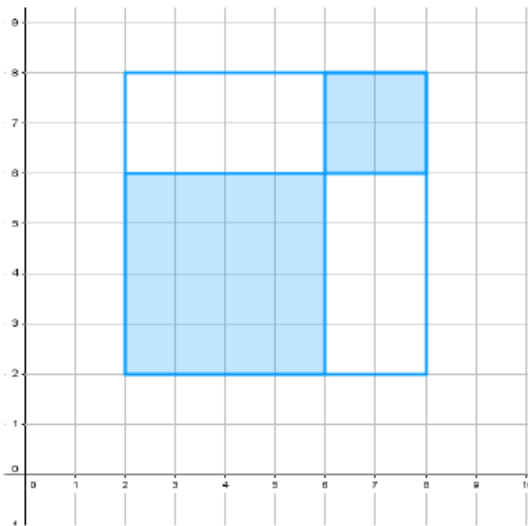
Module 1  $\frac{73}{100}$

Module 2  $\frac{23}{35}$

Module 3  $\frac{40}{60}$

In which module did Jason perform the best?

8. This design has been produced on a coordinate grid.



(a) What fraction of the shape is blue?

(b) What percentage of the shape is blue?

9. This spinner is a regular pentagon split into five equal sections. Complete the three statements to describe the probability of the spinner landing on a shaded section.



As a fraction the probability of the spinner landing on a shaded section is — .

As a decimal the probability of the spinner landing on a shaded section is 0.

As a percentage the probability of the spinner landing on a shaded section is .....%

## 5.2 Expressing one number as a percentage of another

Writing one number as a percentage of another number is a common, everyday necessity. For example, it's used for calculating percentage scores on tests or working out costs on a project.

### Example

A test is out of 50. Beth scores 47. What percentage score did she get?

We can show the score as a fraction:  $\frac{47}{50}$

Then scale the fraction to make denominator 100, or type into a calculator:

$$\frac{47}{50} = \frac{94}{100} = 0.94 = 94\%$$

So Beth scored **94%**.

## Fractions, decimals and percentages

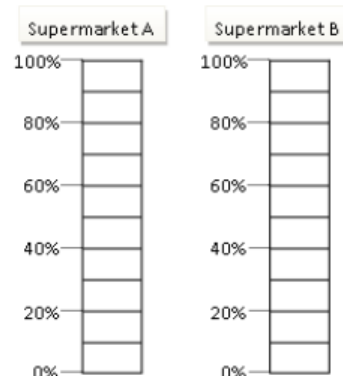
### Practice sheet 2

1. In a taste test some customers were asked to try two different orange drinks and say which they thought was the supermarket's own brand.

- (a) Use the results to fill in the bars on the diagram opposite.

The number of people who:	Supermarket A	Supermarket B
Correctly identified the supermarket orange drink	72	30
Didn't identify the supermarket drink.	48	10
Totals	<b>120</b>	<b>40</b>

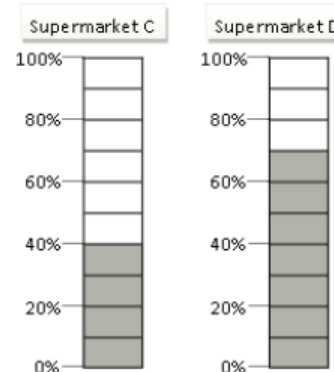
The percentage of customers who correctly identified the own brand orange drink.



- (b) These are the results from the same taste test from Supermarkets C and D. Use the information in the diagram to complete these statements.

..... out of 80 customers in supermarket C correctly identified the own brand orange.

77 out of ..... customers in supermarket D correctly identified the own brand orange.



2. Explain how you know that 20% of the shape below is shaded.



3. A teacher explains that  $\frac{1}{7}$  is 14% to the nearest 1%.

A student says “does that mean that  $\frac{2}{7}$  is 28% to the nearest 1%?”.

Show why the answer to the student’s question is NO.

4. A consumer magazine did a telephone survey. People were asked to say which supermarket they used. They were then asked to rate how happy they were with their shopping experience. The results for three supermarkets are given below.

Happiness rating	Supermarket A	Supermarket B	Supermarket C
Excellent	65	52	40
Good	35	29	18
Poor	10	19	2
Very poor	0	0	0
Totals	<b>110</b>	<b>100</b>	<b>60</b>

For each supermarket, work out the percentage of customers who rated the experience excellent. Place these in order, starting with the supermarket with the highest percentage of its customers rating it as excellent.

5. The manager of coffee shop monitors the drink sales in the shop. The sales are shown below:

Answer the questions below. Give your answers to the nearest 1%

- (a) What percentage of the total sales are teas?  
 (b) What percentage of the total takings was for tea?

	Price	Sales	Takings
Caffe Latte	£2.25	135	£303.75
Cappuccino	£2.25	98	£220.50
Vanilla Latte	£3.45	94	£324.30
Americano	£2.00	160	£320.00
Espresso	£1.70	55	£93.50
Tea	£1.90	103	£195.70
		645	£1,457.75

6. A college is considering offering some new courses. A sample of prospective students are asked to choose one course they would like to do. The results are given below.

- (a) Which course was chosen by 20% of the female students?
- (b) What percentage of the male students chose Art and Design?
- (c) What percentage of the students choosing Applied Science were female?

Course	People showing an interest		
	Male	Female	Total
Art and Design	12	53	65
Sport Studies	53	22	75
Applied science	22	28	50
Creative Media	42	48	90
Construction	41	9	50
Public Services	30	40	70
<b>Totals</b>	<b>200</b>	<b>200</b>	<b>400</b>

Answers: Page 73



### 5.3 Percentage change

For support with this section, have a look at Hegarty clips 97.

It is possible to look at the difference between two numbers and work out the percentage increase or the percentage decrease. This is known as percentage change.

Percentage change is calculated by dividing the difference between the two amounts by the original amount, i.e.:

$$\text{Percentage Change Formula} = \frac{\text{Old Number} - \text{New Number}}{\text{Old Number}} \times 100$$

#### Example

A painting is bought for £200 and later sold for £250. What is the percentage profit?

The difference =  $250 - 200 = 50$

Change/original =  $50/200 = 25/100 = 0.25 = 25\%$

### Percentages, fractions and decimals

#### Practice sheet 3

1. George's wage in 2000 was £4.50 per hour. In 2015 his wage was £6.00 per hour. Calculate the percentage increase in George's wage from 2000 to 2015. Give your answer to 1d.p.

2. Here is some data about two companies

Profits	Company A	Company B
2000	£12million	£92,000
2015	£16million	£140,000

(a) Work out the percentage increase in profits from 2000 to 2015 for company A. Give your answer to 1 d.p.

(b) Which company has seen the greater increase in profits from 2000 to 2015?

3. According to data from the website MeasuringWorth.com the average annual salary in 1975 was £2,291.00. In 2014 the average annual salary was £25,028.60. According to data from the Nationwide Building Society, the average house price in 1975 was £11,000 whilst in 2014 the average house price was £198,000.

(a) Calculate the percentage increase in the average annual salary from 1975 to 2014. Give your answer to the nearest 1%

(b) Calculate the percentage increase in average house price 1975 to 2014. Give your answer to the nearest 1%

4. 30 girls, 40 boys and 130 adults are members of a photography club. After some adverts in schools, 50 more children join the club. The girls now make up 20% of the total membership.

How many boys are there in the club?

5. A BBC news report used some figures from DairyCo in its story.

The figures stated that:

- A four pint carton of milk from a retailer was £1.45.
- From the £1.45, £1.07 was given to the processor.
- The processor then gave 58p to the farmer of the farm where the milk was produced.

(a) (i) Work out how much the retailer gets to keep from the £1.45.

(ii) Work out how much the processor get to keep from the £1.45.

(b) What percentage of the £1.45 goes to the farmer?

(c) It was reported that it costs the farmer 61p to produce four pints of milk. How much does the farmer lose on each four pints?

(d) What is the percentage loss on four pints of milk for the farmer? Give your answer to 1d.p.

Answers: Page 74

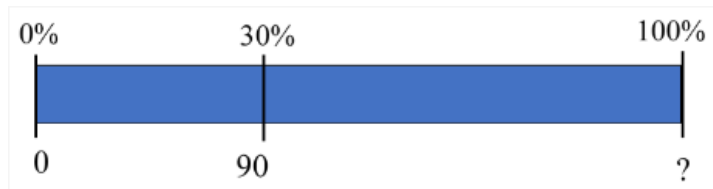
## 5.4 Reverse percentages

For support with this section, have a look at Hegarty clips 96.

Reverse percentages helps to solve questions like:

30% of a number is 90. What is the number?

### Example



If 30% of a number is 90  
 10% of the same number is 30  
 and 1% of the same number is 3.  
 100% of the number is 300.  
 The missing number is **300**.

### Example

**How much is £100 reduced by 20%?**

£100 reduced by 20% is

$$£ \boxed{100} - £ \boxed{20} = £ \boxed{80}$$



- A decrease of 20% means that each £1 becomes £0.80
- So to find the result of a 20% drop, you can multiply the original amount by 0.80
- $£100 \times 0.80 = £80$
- 0.80 is called a multiplier
- Multiplier for percentage decrease is found by taking away 20% (reduction) from 100% (original amount) and converting to a decimal number

### Example

**How much is £100 increased by 25%?**

£100 increased by 25% is

$$£ \boxed{100} + £ \boxed{25} = £ \boxed{125}$$

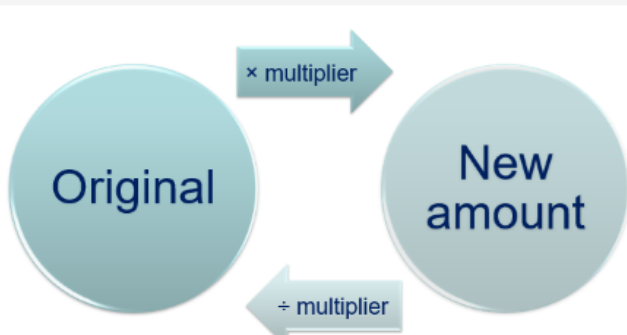


- An increase of £25% means that each £1 becomes £1.25
- So to find the result of 25% increase, you can multiply by 1.25
- $£100 \times 1.25 = £125$
- 1.25 is a multiplier
- The multiplier for percentage increase is found by adding 25% (increase) to 100% (original amount) and converting to a decimal number

### Example

A car is on sale for £14 250 after 5% reduction was applied due to high milage.

**What was the price of the car before the discount was applied?**



Fill in the blanks to answer the question.

The multiplier for 5% reduction is 0.

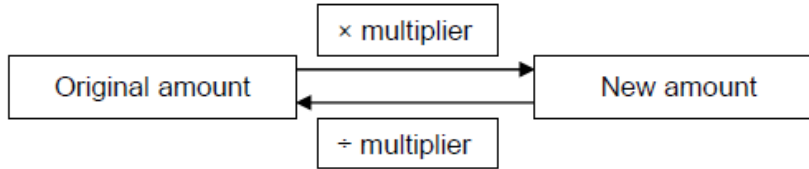
The original price x  = £14,250 (new amount)

£  ÷  = Original price

The original price is £

## Reverse percentages

### Practice sheet 1



1. 25% of a number is 100. What number is it?
2. 35% of a number is 1050. What is 65% of the number?
3. A jacket cost £96 in a 20% off sale. What was the original price?
4. A bed cost £360 in a 25% off sale. What was the original price?
5. A price of a train fare is increased by 4% to £26. What was the original price?
6. A local restaurant increased all prices by 2.5%. The new price of a steak and chips is £15.99. How much did it cost before the price increase?
7. A holiday costs £1075 including VAT at 20%. What is the price of the holiday without VAT?
8. Terry sells an antique ring in an auction for £580 which is 2% higher than the average price of a similar item sold privately. What is the price of a similar antique ring sold privately?
9. Jenny takes 2 hours and 12 minutes to complete her race. This is 10% longer than the winner took. How long did the winner take to complete the race?
10. Every seat in the theatre is full; there are 500 people in the audience. The presenter says that this is exactly 40% more than in the previous day's show.  
Explain why the presenter is incorrect.

Answers: Page 75

## 5.5 Repeated percentage change

For support with this section, have a look at Hegarty clips 91-92.

**Interest** is money that is paid regularly at a particular percentage, usually when money has been lent or borrowed. For example, a bank will give its customers interest to reward them for saving money with them, but it will also charge interest to anyone who has borrowed money from them.

### Simple interest

As the name suggests, simple interest is a quick way of calculating interest. Simple interest is worked out by calculating the percentage amount and multiplying it by the number of periods that the money will be invested for.

#### Example

Calculate the interest on borrowing £150 for 3 years if the simple interest rate is 7.5% per year.

To work out the amount of interest for 1 year calculate 7.5% of £150 and multiply by the number of years to get the interest for 3 years:

$$\begin{aligned}7.5\% \text{ of } £150 &= 150 \times 0.075 = £11.25 \\ £11.25 \times 3 &= £33.75\end{aligned}$$

So, the amount of simple interest on borrowing £150 for 3 years is **£33.75**, which makes the total amount payable £183.75.

### Compound interest

Compound interest means that each time interest is paid onto an amount saved or owed, the added interest also receives interest from then on.

#### Example

Calculate the interest on borrowing £150 for 3 years if the compound interest rate is 7.5% per year.

Increase of 7.5% gives a multiplier of 1.075 (107.5%)

To work out the amount of an interest for year 1 calculate 7.5% of £150:

$$\text{Year 1 } £150 \times 1.075 = £161.25$$

To work out the amount of interest for year 2 calculate 7.5% of year 1 increased amount

$$\text{Year 2 } £161.25 \times 1.075 = £173.34$$

To work out the amount of interest for year 5 calculate 7.5% of year 2 increased amount

$$\text{Year 3 } £173.34 \times 1.075 = \mathbf{£186.34}$$

So the amount of compound interest on borrowing £150 for 3 years is  $186.34 - 150 =$  **£36.34**, which makes the total amount payable £186.34.

When working with a calculator, it is much quicker to go to the final answer in one step:  
 $£150 \times 1.075 \times 1.075 \times 1.075 = £150 \times 1.075^3 = \mathbf{£186.34}$

### Practice sheet - Repeated percentage change

1. Louisa invests £12,000 for 3 years at 3% compound interest per year.

How much interest will Louisa earn after 3 years?

2. Ed leaves £5,000 in the bank for 4 years at 2% compound interest per year.

What is the total amount Ed will have in the bank after 4 years?

3. Amanda invests £7,000 at 2.5% compound interest per year.

How many years will it take for the investment to exceed £8,000?

4. Abbie buys an antique watch for £2,300.

The watch decreases in value by 3.5% every year.

How much will the watch be worth in 3 years?

5. Richard buys a motorbike for £9,000.

The value of the bike decreases by 7.5% every year.

How much will the bike be worth in 4 years when he is planning to trade it for a new one. (Give your answer to the nearest £100)

6. Steve bought a painting for £15,000. A year later the painting was worth £14,400.

(a) Calculate the percentage decrease in the value of the painting.

(b) If the value of the painting decreases at the same rate, how much will it be worth after a further 7 years?

7. Alex bought her house for £230,000.

She lived in her house for 4 years and then put it up for sale.

Each of the 4 years, the house increased its value by 4.5%.

How much was the house worth after 4 years when Alex decided to sell it? (Give your answer to the nearest £1000)

Answers: Page 78

## 6. Probability

### 6.1 Outcomes

For support with this section, have a look at Hegarty clips 351-355, 358-367.

#### Example

In the 18th century Jean le Rond d'Alembert, a famous mathematician, discussed a game of heads and tails:

A player wins if they get heads at least once on two tosses of a coin.

When flipping two coins there are 4 possible outcomes:

HH HT TH TT

Outcomes are all the possibilities of what could happen.

With a fair coin, each of these outcomes are equally likely.

As three of the four equally likely outcomes have at least one head (a win).

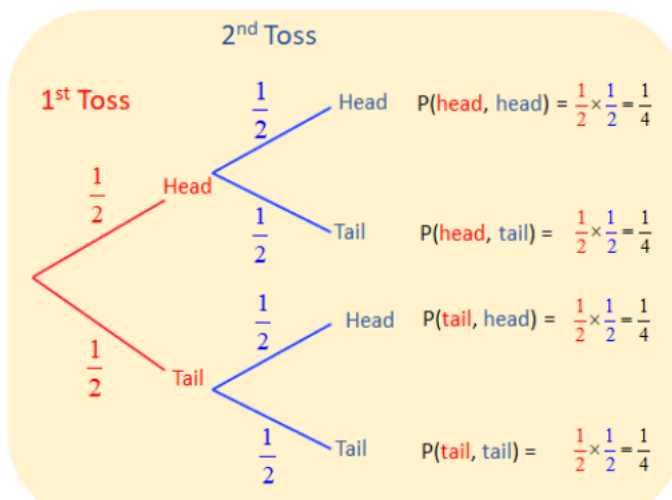
HH HT TH TT

The probability of winning at the game is  $\frac{3}{4}$

Sometimes when you list all the outcomes you are not sure if you have got them all. Putting all your possibilities in a tree diagram can help to list all the outcomes.

#### Example

Tree diagram to show the probabilities when a coin is tossed twice.





## Outcomes

1. Anna has been invited to a wedding and can choose one starter, one main course and one dessert from the following menu:

<b>Starter</b>	<b>Main</b>	<b>Dessert</b>
Soup	Chicken	Ice cream
Nachos	Vegetarian	Cheese

- a. Draw a tree diagram to show all the possible outcomes she can choose.
- b. Anna decides to choose each course at random. What is the probability she chooses both Soup and the Vegetarian option?
2. A model of car comes with or without air conditioning, with or without a sunroof and in four colours – blue, grey, black or white. Cars with each feature are made in equal numbers.
- a. Draw a tree diagram to show all the possible choices of cars.
- b. Ranjit wants a new grey or white car of this model with a sunroof, but does not mind about air conditioning. The local garage is getting a car of this model next week but it will be chosen at random from all the cars produced. What is the probability that Ranjit will want the car at the local garage?
3. Computers use binary numbers. A binary number is made up of only 0s and 1s. The one digit binary numbers are 0 and 1 and the two digit binary numbers are 10 and 11 as binary numbers do not start with 0.
- c. How many four digit binary numbers are there?
- d. How many six digit binary numbers are there?

Answers: Page 79

## 6.2 Relative frequency

For support with this section, have a look at Hegarty clips 356-357.

Murphy's Law says that if you drop a piece of toast, it is more likely to land butter side down.

How could you test this?

**Frequency** is how many times an event occurs.

**Relative frequency** =  $\frac{\text{number of times an event occurs}}{\text{total number of trials}}$

You can estimate probability by working out relative frequency.

### Example

A piece of buttered toast was dropped 50 times and landed butter side down 30 times.

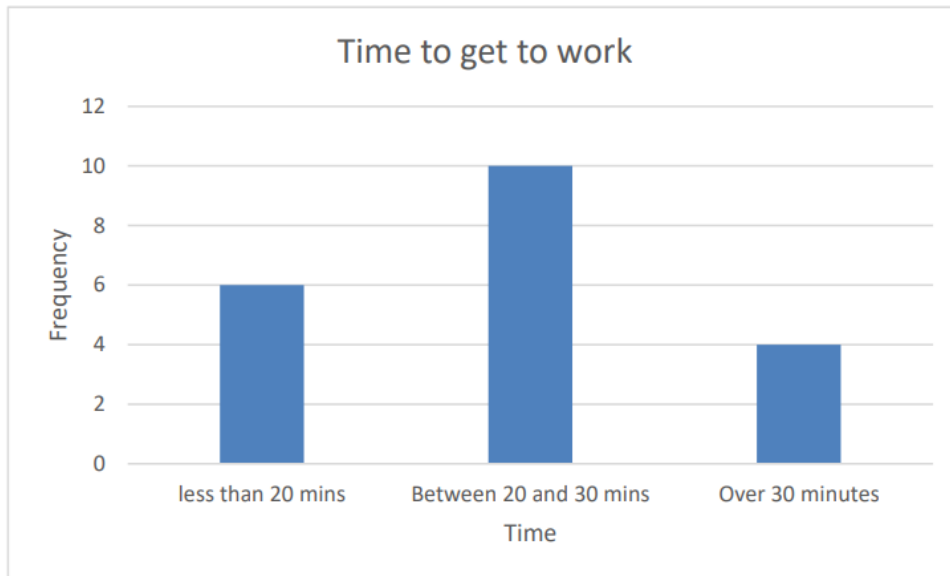
The relative frequency of the toast landing butter side down is  $30/50$

So, an estimate of the probability of the toast landing butter side down is  $30/50 = \mathbf{3/5}$



## Questions: Working out relative frequencies

1. Two athletes have raced each other 30 times. Tom has won 16 times and Alex has won 14 times. Estimate the probability that Alex wins the next race.
2. Folklore says if it rains on St Swithin's Day, July 15<sup>th</sup>, it will rain for the next forty days. In the last 20 years in a town it has rained 6 times on St Swithin's Day. Estimate the probability it will rain on July 15<sup>th</sup> next year.
3. Chris's football team have played 44 matches this season and won 10, drawn 13 and lost 21. Estimate the probability that the team lose their next match.
4. A local weather expert forecast the weather over the month of July. He was correct on 15 occasions in the first 30 days. Estimate the probability his forecast on the last day of July will be correct.
5. Kate drives to work every day. The bar chart shows how often travelling to work took her over the last 4 weeks.



Estimate the probability it takes her over 30 minutes to get to work the next time she drives in.

Answers: Page 81

### 6.3 Two-way tables

For support with this section, have a look at Hegarty clips 422-424.

Putting results into a table helps when finding probabilities.

#### Example

	Passed	Failed	Total
Homework	42	16	58
No homework	19	13	32
Total	61	29	90

For the students that did homework, the proportion who passed was  $\frac{42}{58} = 0.724$

For the students that didn't do homework, what was the proportion who passed?

Total who didn't do homework: 32

No homework and passed: 19

Proportion = **19/32**

#### Example

The table shows the results of test to see whether people have a particular disease

Test results

Person's situation	Positive	Negative	Total
Has disease	291	9	300
Doesn't have disease	22	678	700
Total	313	687	1000

Altogether **1000** were tested. Out of the **300** people with the disease, **291** received a positive result. This means **9** people with disease got the wrong result.

There were **700** people who didn't have the disease, of these **678** got the correct result from the test and **22** got an incorrect result.

## Was this an effective test?

Test results

Person's situation	Positive	Negative	Total
Has disease	291	9	300
Doesn't have disease	22	678	700
Total	313	687	1000

Probability of test being right for those who have the disease =  $\frac{291}{300}$

Probability of test being right for those who don't have the disease =  $\frac{678}{700}$

Probability of test being wrong for those who have the disease =  $\frac{9}{300}$

Probability of test being wrong for those who don't have the disease =  $\frac{22}{700}$

Probability of test being right =  $\frac{291+678}{1000} = \frac{969}{1000}$

Probability of test being wrong =  $\frac{9+22}{1000} = \frac{31}{1000}$

There is a very high probability that the test is correct.

## Questions: Two-way tables

1. The table below shows the gender and political party of MPs who were voted into Parliament.

	Conservative	Labour	Lib Dem	Others	Total
Men	257	169	50	23	499
Women	49	89	7	6	151
Total	306	258	57	29	650

An MP was chosen at random from this parliament

- What is the probability a woman is chosen?
- What is the probability a Conservative MP is chosen?
- What is the probability a female Liberal Democrat is chosen?
- What is the probability someone who is not a Conservative is chosen?

2. Blood grouping is defined by the ABO system. The four main blood groups are O, A, B and AB and these can be either positive or negative.

a) Draw a tree diagram to show all the possibilities for the type of blood which there are.

The table shows the percentage of people in the UK with each blood group.

	O	A	B	AB	Total
Positive	37	35	8	3	83
Negative	7	7	2	1	17
Total	44	42	10	4	100

a) What is the probability that a person chosen at random has B negative blood?

b) If you know a person has blood of type A but you do not know if they are positive or negative, what is the probability they have positive blood?

c) What percentage of people in the UK have positive blood?

3. 100 car divers were sampled to see if they passed their theory and practical tests first time. The results are in the table.

		Theory		
		Pass	Fail	Total
Practical	Pass	24	21	45
	Fail	32	23	55
	Total	56	44	100

a) Estimate the probability that someone will pass both tests first time

b) Estimate the probability that someone will pass one test first time but fail the other one first time

c) Sam admits he failed his theory test first time. Estimate the probability that he will pass his practical first time

4. A new drug was tested by giving 60 people the drug and 80 people a placebo. A control group consisting of 100 people were given no treatment. The results are shown in the table

	Drug	Placebo	Control	Total
Improvement	47	62	35	144
No improvement	13	38	65	116
Total	60	80	100	240

- a) A person is selected at random from the 240 people represented in the table. What is the probability he was either in the control group or was given the placebo?
- b) Use the data in the table to estimate the probability that someone who is given the drug will improve.
- c) Use the data in the table to estimate the probability that someone who has improved was given the drug.

Answers: Page 82

## 7. Proportional reasoning

For support with this section, have a look at Hegarty clips 339-341.

When someone travels to another country, they often need to buy currency. Exchanges show their rates. If you know what £1 buys you can work out how much you will get for other amounts.

### Example

If £1 = \$1.23 then:

$$£2 = 2 \times \$1.23 = \mathbf{\$2.46}$$

$$£7 = 7 \times \$1.23 = \mathbf{\$8.61}$$

Conversions are used in a similar way when changing between measurements.

### Example

We use the fact that 5miles = 8 km to help us work out other distances.

$$10 \text{ miles} = 2 \times 5 \text{ miles} = 2 \times 8\text{km} = \mathbf{16\text{km}}$$

$$32\text{km} = 4 \times 8\text{km} = 4 \times 5 \text{ miles} = \mathbf{20 \text{ miles}}$$

When working with speed the units help to explain how far the object travels in a set period of time.

### Example

10 metres per second (10m/s), means the object moves 10 metres every second.

At 10m/s after 3 seconds the object will move  $3 \times 10\text{m} = \mathbf{30\text{m}}$ .

We sometimes use other units like kph, kilometres per hour and mph miles per hour but the principle is the same, how far the object has moved, 1km or 1mile, in what time (1hour).

The same proportional reasoning ideas you used with the currency and speed problems can be used to work out which offer is best. One way to do this is to buy the same amount using both offers and compare the cost. Another way to compare offers is by using unit pricing.

### Example

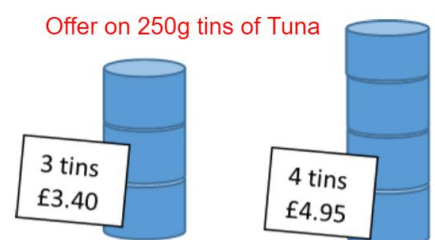
You can buy 3 tins for £3.40 or 4 tins for £4.95.

Method 1: work out the cost of the same number of tins

How much are 12 tins in packs of 3?  $4 \times £3.40 = £13.60$

How much are 12 tins in packs of 4?  $3 \times £4.95 = £14.85$

Buying in **packs of three is cheaper** and the best buy.





Method 2: work out the cost one tin

For the 3 pack the cost of one tin is  $\pounds 3.40 \div 3 = \pounds 1.13$  (2d.p.)

For the 4 pack the cost of one tin is  $\pounds 4.95 \div 4 = \pounds 1.24$  (2d.p.)

Comparing these prices, the **3 pack is the best buy** because each tin costs less.

The cost of one tin is called the unit price, the tin is the unit.

## Proportional reasoning

### Practice sheet

1. A lorry is travelling at a constant speed of 40kph.

How many kilometres will it travel in:

- (a) 3 hours  
(b) 2½ hours  
(c) 15 minutes  
(d) 12 minutes

2. Use these conversions:

$$\pounds 1 = \$1.32, \pounds 1 = \text{€}1.06$$

to answer below questions.

- (a)  $\pounds 20 = \$$ \_\_\_\_  
(b)  $\pounds 36 = \$$ \_\_\_\_  
(c)  $\pounds 78 = \text{€}$ \_\_\_\_  
(d)  $\pounds$ \_\_\_\_ = \$15.84  
(e)  $\pounds$ \_\_\_\_ = €44.52

3. Fazal and Bashir buy the same MP3 player.

Fazal pays  $\pounds 95$

Bashir pays  $\text{₹}8250$

Key  $\text{₹}$  - Indian Rupees

At the time  $\pounds 1 = \text{₹}85$

- (a) How much did Bashir pay in  $\pounds$ ?  
(b) Who got a better deal?

4. The price of a litre of petrol in UK and in the Republic of Ireland are both shown below.



€1.32 per litre of petrol



£1.22 per litre of petrol

The exchange rate at the time the prices were compared is  $\pounds 1 = \text{€}1.10$

- (a) Which country has the cheapest petrol?  
(b) What is the difference in the price of petrol?

5. 5 text books cost £29, what would 9 of these text books cost?
6. 30 pens cost £4.50
- (a) Find the cost of 40 pens.
  - (b) How many pens could you buy with £100?
7. A car uses 40 litres of fuel to travel 140 km.
- (a) How much fuel would the car need to travel 98 km? (Answer to the nearest litre)
  - (b) How far will the car travel on a full tank (75 litres) of fuel? (Answer to 1 decimal point)
8. A customer wants to buy some snack bars. The shop sells a pack of 5 snack bars which cost £2.55, or a pack of 8 snack bars which cost £3.92.
- (a) What is the cost of each bar in the 5 pack?
  - (b) What is the cost of each bar in the 8 pack?
  - (c) Which pack is the best value for money?
9. A printer can print 12 pages per minute (ppm).  
How long will it take to print 100 page document?
10. Anya is comparing her GCSE marks.
- In history she scored 56 out of 80  
In geography she scored 73 out of 100  
In which subject did she get a higher percentage?
11. 5 miles = 8 km  
A car is travelling at 56kph  
What is this speed in mph?
12. 5 miles = 8 km  
Angelina says her car can travel 400 miles on 30 litres of fuel.  
Brett says his car can travel 600 km on 30 litres of fuel.
- (a) How many km can Angelina's car travel on 30 litres of fuel?
  - (b) How far can Brett's car travel on 8 litres of fuel?

Answers: Page 84

## 8. Algebra

### 8.1 Solving equations

For support with this section, have a look at Hegarty clips 177-186.

#### Two-step equations

1. Add or subtract to leave only the variable term on one side of the '='.
2. Multiply or divide to find the solution.
3. Check your solutions.

#### Example

Solve  $3x + 5 = -16$

$$\begin{array}{r} 3x + 5 = -16 \\ -5 \quad -5 \quad \text{Subtract} \end{array}$$

$$3x = -21$$

$$\begin{array}{r} \frac{3x}{3} = \frac{-21}{3} \quad \text{Divide} \end{array}$$

$$x = -7$$

$$3(-7) + 5 = -16 \quad \text{Check}$$

Final answer:  $x = -7$

#### x on both sides

1. If you have brackets, expand first.
2. Add or subtract terms to get the variable (usually x) on one side of the '=', and any numbers on the other side.
3. Multiply or divide to find the solution.
4. Check your answer.

#### Example

$$5(x + 2) = 3x - 8$$

$$5x + 10 = 3x - 8$$

$$5x - 3x + 10 = 3x - 3x - 8$$

$$2x + 10 = -8$$

$$2x + 10 - 10 = -8 - 10$$

$$2x = -18$$

$$2x/2 = -18/2$$

$$x = -9$$

Check:

$$5(-9 + 2) = 3(-9) - 8$$

$$-35 = -35$$

Final answer:  $x = -9$

### Practice Questions: Solving equations

1) Solve the following equations.

(a)  $2x + 3 = 9$

(b)  $3w - 1 = 14$

(c)  $7y + 2 = 30$

(d)  $5x + 20 = 35$

(e)  $6c - 12 = 48$

(f)  $8m - 4 = 20$

(g)  $7w + 13 = 90$

(h)  $12p - 18 = 30$

(i)  $9w - 5 = 67$

2) Solve the following equations.

(a)  $4x + 1 = 2x + 7$

(b)  $5x + 4 = 3x + 16$

(c)  $2x + 8 = x + 12$

(d)  $7x + 1 = 2x + 46$

(e)  $6x - 3 = 2x + 13$

(f)  $9x - 10 = 7x + 24$

3) Solve the following equations.

(a)  $5(x + 3) = 3(x + 9)$

(b)  $8(x - 1) = 4(x + 3)$

(c)  $3(x + 13) = 10(x - 1)$

Answers: Page 86

## 8.2 Solving Inequalities

For support with this section, have a look at Hegarty clips 269-271.

To solve inequalities, we follow exactly the same steps as solving equations, we just have a different symbol in the middle (an inequality sign rather than '=').

### Example

*Problem:*  $2x - 5 < 1$

*Solution:*  $2x - 5 + 5 < 1 + 5$       **+5**  
 $2x < 6$   
 $\frac{2x}{2} < \frac{6}{2}$   
 $x < 3$       **÷ 2**  
Final answer:  **$x < 3$**

### Example – x on both sides

$3(2x - 4) + 6 > 18 + 2x$       {Remove brackets}  
 $6x - 12 + 6 > 18 + 2x$       {Simplify}  
 $6x - 6 > 18 + 2x$       {Subtract 2x from both sides}  
 $6x - 6 - 2x > 18 + 2x - 2x$   
 $4x - 6 > 18$       {Add 6 to both sides}  
 $4x - 6 + 6 > 18 + 6$   
 $4x > 24$       {Divide both sides by 4}  
 $\frac{4x}{4} > \frac{24}{4}$   
 $x > 6$

We must be careful if there is a negative in front of the x term. If you **divide by a negative number**, you need to **swap the direction of the inequality sign**.

### Example

$-3x + 5 \leq -16$   
 $-5 \quad -5$       **Subtract**  
 $-3x \leq -21$   
 $\frac{-3x}{-3} \geq \frac{-21}{-3}$       **Divide by -3, reverse inequality**  
 $x \geq 7$

## Practice Questions: Solving inequalities

1) Solve each of the inequalities:

(a)  $x + 4 > 9$     (b)  $x - 3 < 2$     (c)  $2x \leq 14$     (d)  $8x < 16$

2) Solve each of the inequalities:

(a)  $2x + 1 \leq 9$     (b)  $3x - 5 > 16$     (c)  $4x + 8 < 32$     (d)  $5x - 2 \geq 68$

3) Solve each of the inequalities:

(a)  $4x + 3 > 2x + 11$     (b)  $x + 1 \geq 3x - 18$

(c)  $13x - 12 < 3x + 13$     (d)  $7x - 5 \geq 3x + 11$

Answers: Page 87

### 8.3 Solving simultaneous equations

For support with this section, have a look at Hegarty clips 190-193.

Here we will consider solving 2 simultaneous equations using elimination.

#### Example

$$\begin{aligned}2x &= -3y + 16 \\5x - 4y &= -6\end{aligned}$$

1: "Line up" the variables.

$$\begin{aligned}2x + 3y &= 16 \\5x - 4y &= -6\end{aligned}$$

2: Determine which variable to eliminate. Make the coefficients opposites.

$$\begin{aligned}2x + 3y &= 16 & \rightarrow & 10x + 15y = 80 \\5x - 4y &= -6 & \rightarrow & -10x + 8y = 12\end{aligned}$$

x 5  
x -2

3: Add straight down. (One variable should be eliminated.)

4: Solve the resulting equation.

$$\begin{array}{r}10x + 15y = 80 \\+ -10x + 8y = 12 \\ \hline23y = 92 \\ y = 4\end{array}$$

÷ 23

5: Substitute this result into either of the original equations.

6: Solve for the variable to find the ordered pair solution.

$$\begin{aligned}2x + 3(4) &= 16 \\x &= 2\end{aligned}$$

Final answer:  
**x = 2, y = 4**

7: Check the solution in both original equations.

### Practice Questions: Solving simultaneous equations

Solve the following by elimination:

(a)  $6x + y = 18$   
 $4x + y = 14$

(b)  $4x + 2y = 10$   
 $x + 2y = 7$

(c)  $9x - 4y = 19$   
 $4x + 4y = 20$

(d)  $9x - 4y = 59$   
 $2x - y = 12$

(e)  $2x + 8y = 43$   
 $x + 3y = 18$

(f)  $6x + 3y = 45$   
 $2x - 2y = 12$

(g)  $3x + 8y = 31$   
 $5x + 3y = 31$

(h)  $7x - 15y = 2.5$   
 $3x - 2y = 5.5$

(i)  $3x + 2y = 53$   
 $2x + 5y = 72$

Answers: Page 88



# Answers

## 1. Data tables, averages and spread

### Data tables, averages and spread: Practice sheet 1 - Solutions

1. Himanji made a list of her homework marks for the last ten weeks:

14 15 15 15 14 13 11 12 14 15

(a) Write down the mode of her homework marks

(b) Work out her mean homework mark

(a) The mode is the most common mark. This is 15

(b) Mean =  $\frac{14+15+15+15+14+13+11+12+14+15}{10} = 13.8$

2. Here is a set of numbers:

8 7 3 5 2

Add a number to ensure you get a median of 6

The number in order are:

2 3 5 7 8

Median is the middle value, in the list above this is 5.

Adding an extra number will give a list 6 numbers. The median of this new list will be the mean of the 3rd and 4th number in the list.

To get a median of 6, 5 must be in 3<sup>rd</sup> position and 7 must be in 4<sup>th</sup> place

This will only happen if new number is 7 or more

3. The test marks for 8 girls and 6 boys are shown:

Girls:        15    13    18    12    17    13    13    18  
Boys:        12    15    19    13    15    7

- (a) Work out the median mark of the boys.
- (b) Work out the range of the girls.
- (c) Are the girls' marks more consistent than the boys' marks? Explain your answer.

(a) In order the marks are

11   12   13   15   15   19

The median is half way between 13 and 15 so, **14**

(b) Range = biggest – smallest

$$= 18 - 12 = 6$$

(c) Range for boys =  $19 - 7 = 12$

The girls' marks are more consistent as there is a smaller range

4. A football team played 7 games. The number of goals scored in each game was:

4    2    1    5    0    1    4

Work out the median number of goals.

In the next game the football team scores 4 goals.

Find the median number of goals scored in the 8 games.

The number of goals in order is

0   1   1   2   4   4   5

So, the median is 2

The number of goals in order is now

0   1   1   2   4   4   4   5

The median is  $\frac{2+4}{2} = 3$

5. Ursula is training to walk up the Machu Picchu trail in Peru. This is 43km long. In four of the five weeks leading up to the walk she has walked

42km      48km      41km      38km

Ursula thinks her mean for the four weeks is more than the length of the trail.

Is she right?

She wants to increase her mean so that it is 45km. How far should she walk on the fifth week?

$$\text{Mean} = \frac{42+48+41+38}{4} = 42.25$$

No, she isn't right

$$\text{Total for 4 weeks} = 42 + 48 + 41 + 38 = 169$$

$$\text{For the mean to be 45, the total for 5 weeks} = 45 \times 5 = 225$$

$$225 - 169 = 56$$

So, she should walk 56km on the fifth day

### Data tables, averages and spread: Practice sheet 2 - Solutions

1. Jane asked 20 people how many chocolate bars they ate last week. The results are shown in the table:

Number of chocolate bars	Number of people
0	4
1	3
2	6
3	1
4	0
5	4
6	1
7	1

- (a) Write down the modal number of chocolate bars eaten.  
(b) Work out the median number of chocolate bars eaten.  
(c) Calculate the mean number of chocolate bars eaten.

(a) Modal number = 2 (the group that has most people in it)

(b) Total number of people = 20

The median will be the average of the 10<sup>th</sup> and 11<sup>th</sup> person

Number of chocolate bars	Number of people
0	4
1	3
2	6
3	1
4	0
5	4
6	1
7	1

$$4 + 3 = 7$$

$$4 + 3 + 6 = 13$$

So the 10<sup>th</sup> and 11<sup>th</sup> person ate 2 chocolate bars

(c) The total number of bars =  $(4 \times 0) + (3 \times 1) + (6 \times 2) + (1 \times 3) + (0 \times 4) + (4 \times 5) + (1 \times 6) + (1 \times 7) = 51$

$$\text{Mean} = 51 \div 20 = 2.55$$

2. The heights of students in a class are measured.  
The results are shown in the table:

Height interval ( h cm)	Number of students
$160 \leq h < 165$	4
$165 \leq h < 170$	7
$170 \leq h < 175$	5
$175 \leq h < 180$	3
$180 \leq h < 185$	1

- (a) Write down the group in which the median lies.  
 (b) Work out an estimate of the mean height of the students.  
 (c) Why is this an estimate?  
 (d) Margret says that she thinks the class has both boys and girls in it. Give a reason why the heights in the class are consistent with Margret's opinion.

(a) Total number of students = 20

So the 10<sup>th</sup> and 11<sup>th</sup> student are in the middle

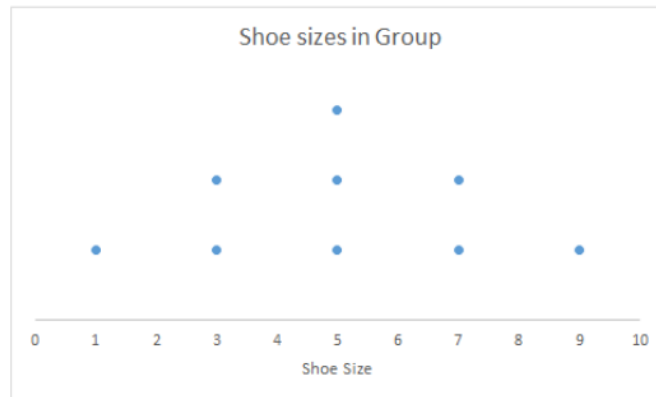
$4 + 7 = 11$ , so the median student is in the second interval  $165 \leq h < 170$ , because both the 10<sup>th</sup> and 11<sup>th</sup> student lie in this interval.

(b) Mean =  $\frac{(4 \times 162.5) + (7 \times 167.5) + (5 \times 172.5) + (3 \times 177.5) + (1 \times 182.5)}{20} = 170$

(c) This is an estimate because the midpoint of each interval has been used and not all the heights would have been this value

(d) There is one height over 180cm, this is likely to be a boy and 4 less than 165cm, these are likely to be girls.

3. A group of nine students marked their shoe sizes on the dot plot below.



a) Find the mode, median and mean shoe size in the group.

b) What do you notice?

c) Two more students add their shoe sizes. They are size 1 and size 9. Mark these on the dot plot.

d) What are the mode, median and mean now?

e) By considering the dot plot, explain why this has happened.

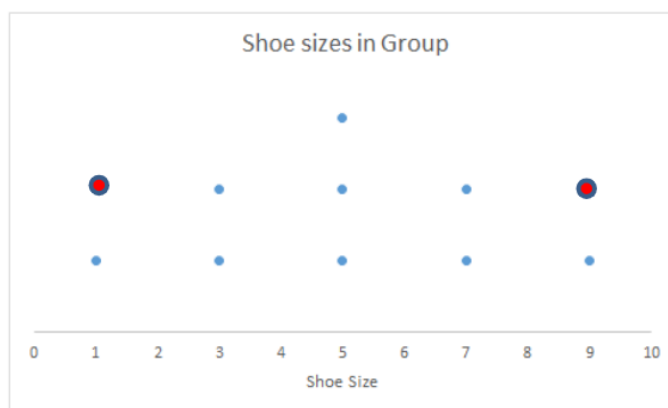
a) Mode = 5 (3 students have size 5)

Median = 5

Mean =  $\frac{1+3+3+5+5+5+7+7+9}{9} = 5$

b) They are all equal to 5

c)



d) Mode = 5 (3 students have size 5)

Median = 5

Mean =  $\frac{1+1+3+3+5+5+5+7+7+9+9}{11} = 5$

e) The dot plot has remained symmetrical. The extra sizes have not affected the mode or median and although we added 1 and 9 to the total of the shoe sizes we then divided by 11 rather than 9 so the mean remained as 5.

4. Each year, more than 150 countries are surveyed for the World Happiness Report.

The main question asked is: "Please imagine a ladder, with steps numbered from 0 at the bottom to 10 at the top. The top of the ladder represents the best possible life for you and the bottom of the ladder represents the worst possible life for you. On which step of the ladder would you say you personally feel you stand at this time?"

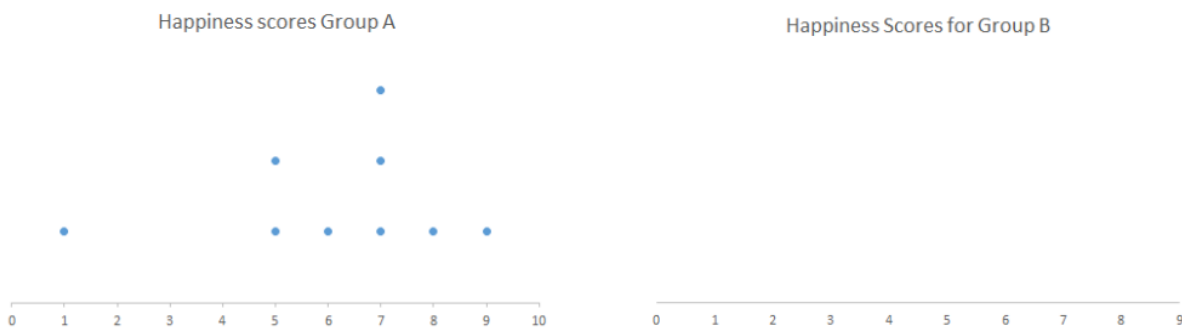
A psychology teacher wants to use this question to compare the happiness levels in their two Year 12 classes. The results are in the table below.

Group A	7	6	5	7	8	1	5	7	9
Group B	8	6	6	5	6	6	7	7	

a) Before you do any calculations, which group do you think is happiest and why?

The dot plot below shows the happiness scores for Group A.

b) Complete the dot plot for Group B.



c) Calculate the mean, median, mode and range for Group A and Group B.

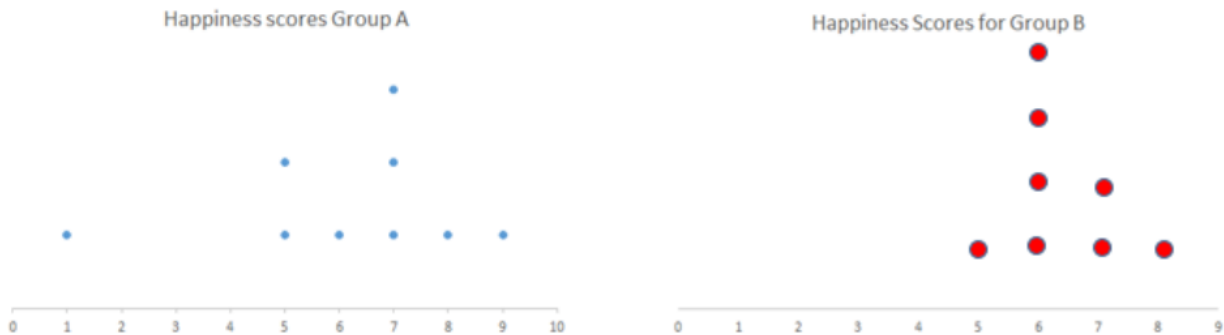
d) Based on your results and by comparing the dot plots, would you change your answer to part (a)? Please explain why.

a) Possible answers

Group A happiest: One person rated their happiness as 9 and no one in Group B gave such a high rating. The most popular rating in Group A was 7 whereas it was only 6 in Group B.

Group B happiest: There were no ratings lower than 5 whereas in Group A one person gave a rating of 1. In Group A, 3 out of 9 gave a score of 5 or below whereas in Group B, only 1 out of 8 gave a score of 5 and it was the lowest rating in the whole group.

b) Completed dot plot for B



c) Group A:

$$\text{Mean} = \frac{1+5+5+6+7+7+7+8+9}{9} = 6.1 \text{ (to 1 d.p.)}$$

$$\text{Median} = 7$$

$$\text{Mode} = 7$$

$$\text{Range} = 9 - 1 = 8$$

Group B:

$$\text{Mean} = \frac{5+6+6+6+6+7+7+8}{8} = 6.4 \text{ (to 1 d.p.)}$$

$$\text{Median} = 6$$

$$\text{Mode} = 6$$

$$\text{Range} = 8 - 5 = 3$$

d) Possible answers:

Group A is happier because the median rating is 7 and the mean is only 0.3 lower than the mean for Group B. The range is bigger though so the levels of happiness are not as consistent as they are in Group B.

Group B is happier because the mean is higher than it is in Group A. The class is more consistently happy than they are in Group B. This is shown by the range being 3 rather than 8.

## 2. Accuracy and estimation

### Answers:

#### Accuracy

1. C. A ruler measures to the nearest mm.
2. B. This number has been rounded to the nearest 1000.
3. a) 2.6      b) 19.5      c) 5.0

(Note: With option c) 5.0, the zero after the decimal point shows that this figure has been rounded to 1 decimal place, not just to the nearest whole number)

4. a) 6.37      b) 14.80      c) 132.46
5. a) 300      b) 4      c) 0.002
6. a) 25,000      b) 0.70      c) 20 (Note - rounding to 1sf also gives 20 as an answer)

#### Estimation

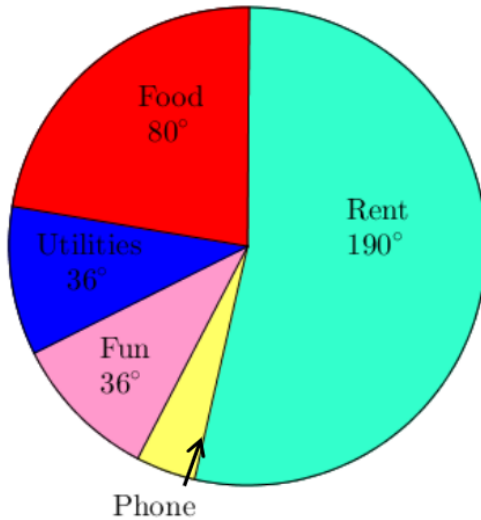
1. A III  
B II  
C IV  
D I
2. a)  $\frac{2 \times 140}{7} = 40$  so III. 40  
b)  $\frac{100}{4} = 25$  so I. 25  
c)  $40 \times 5 \times 2$  (both ways) = 400 miles
3.  $2 \times 2 \times 4 = 16\text{m}^3$



### 3. Interpreting graphs and charts (statistics for life)

#### Solutions - Practice sheet: Statistical diagrams

1. This pie chart shows the monthly budget for a student, Adi.

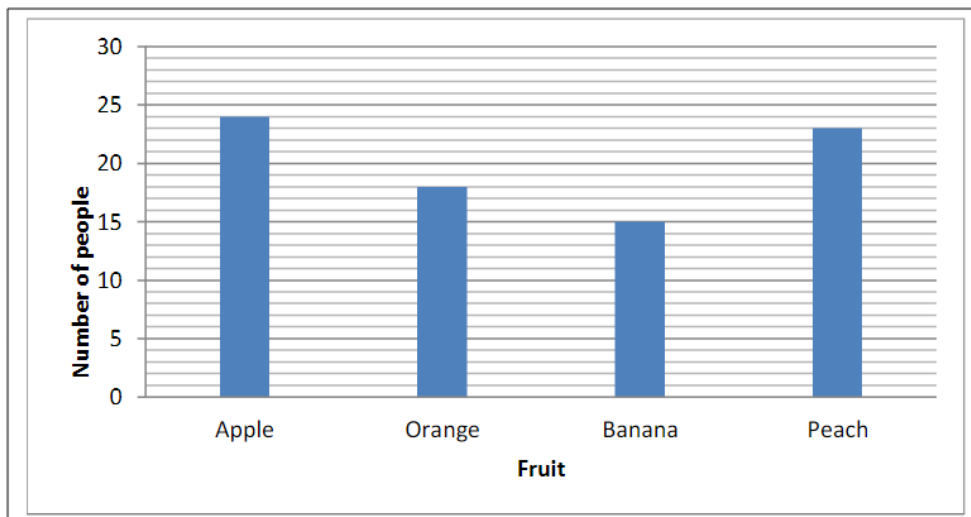


- What is the angle of the sector for "Phone"?
- Adi spends £380 on rent each month. How much does he spend on his phone?

Answer

- The pie should be  $360^\circ$  in total.  
 $360 - 190 - 80 - 36 - 36 = 18^\circ$  so phone should be  $18^\circ$
- If  $190^\circ$  represents £380, then  $1^\circ$  represents  $\frac{380}{190} = \text{£}2$ , so Adi spends  $18 \times \text{£}2 = \text{£}36$

2. A survey was conducted to determine the favourite fruit of 80 people. The bar chart is shown:



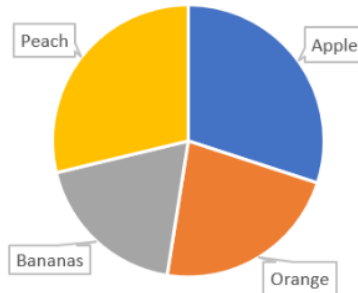
- How many more people preferred apples to bananas?
- Draw and label a pie chart to represent this data.

Answer

- a) 24 people preferred apples, 15 preferred bananas, so  $24 - 15 = 9$  more people preferred apples
- b) The pie needs to represent the 80 people surveyed, so  $360^\circ$  for 80 people, this gives  $\frac{360}{80} = 4.5^\circ$  for each person.

The correct answer should be a pie chart with the following sectors

Apple  $24 \times 4.5 = 108^\circ$   
 Orange  $18 \times 4.5 = 81^\circ$   
 Bananas  $15 \times 4.5 = 67.5^\circ$   
 Peach  $23 \times 4.5 = 103.5^\circ$



5. The pie chart shows the sale of 720 train tickets:  
 There were three times as many senior citizen tickets sold as child tickets. How many child tickets were sold?

Answer

Let  $x$  be the number of child tickets sold, which makes the number of senior tickets  $3x$ .

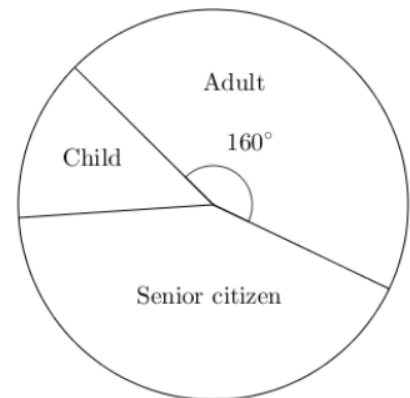
The missing sectors of the pie total  $200^\circ$  ( $360 - 160$ )

So  $4x = 200$  making  $x = 50^\circ$

So the sector for child tickets is  $50^\circ$

If the pie represents 720 tickets, then each  $1^\circ$  slice represents  $\frac{720}{360} = 2$  tickets

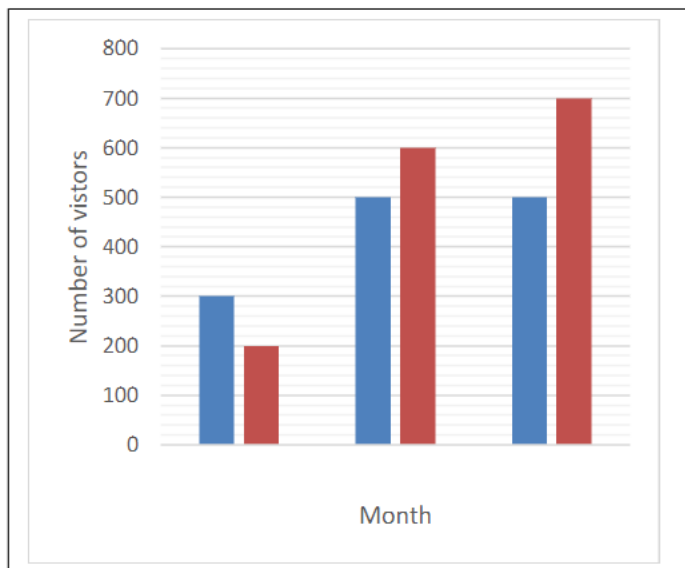
So the number of child tickets is  $50 \times 2 = \underline{100}$



6. The table shows the number of male and female visitors at a garden for three months:

Month	Male	Female
May	300	200
June	500	600
July	500	700

The same information is also shown on the bar chart:



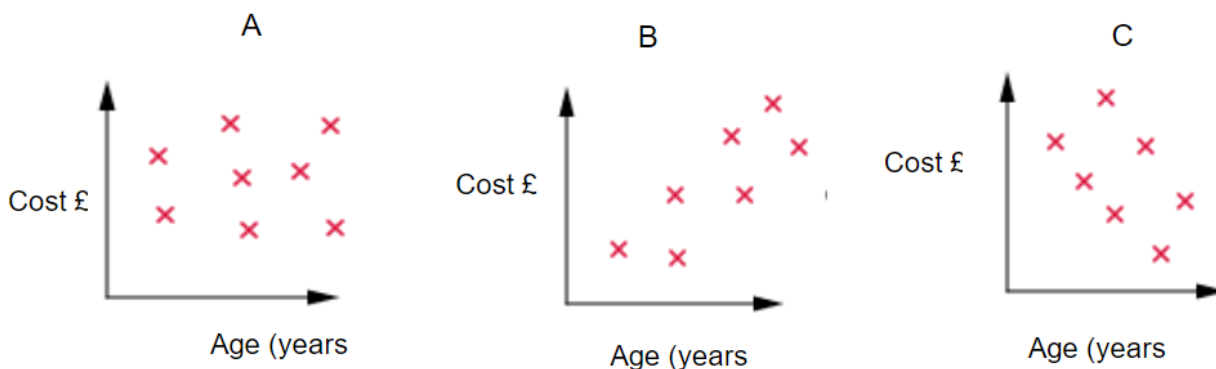
There are three mistakes on the bar chart. What are they?

**Answer** (in any order)

- No months shown on the horizontal axis
- No key to say which colour bar is male and female
- No title on the diagram

### Solutions to Questions: Introduction to scatter diagrams

1. Here are three scatter graphs showing the age of second hand cars and their cost:



Allan says “the older a car, the more it costs”

Jane says “the older a car, the less it costs”

Ranjit says “there is no relationship between the age of a car and its cost”

Match a scatter graph to each statement.

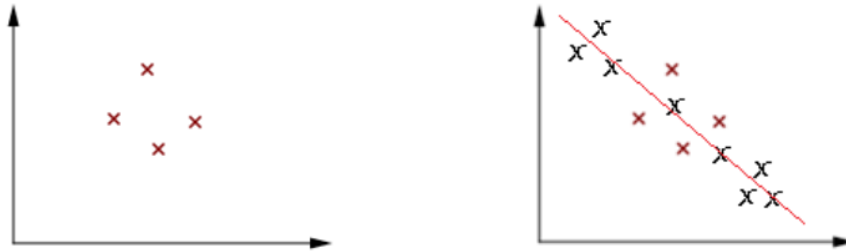
**Answer**

Allan says “the older a car, the more it costs” Diagram B

Jane says “the older a car, the less it costs” Diagram C

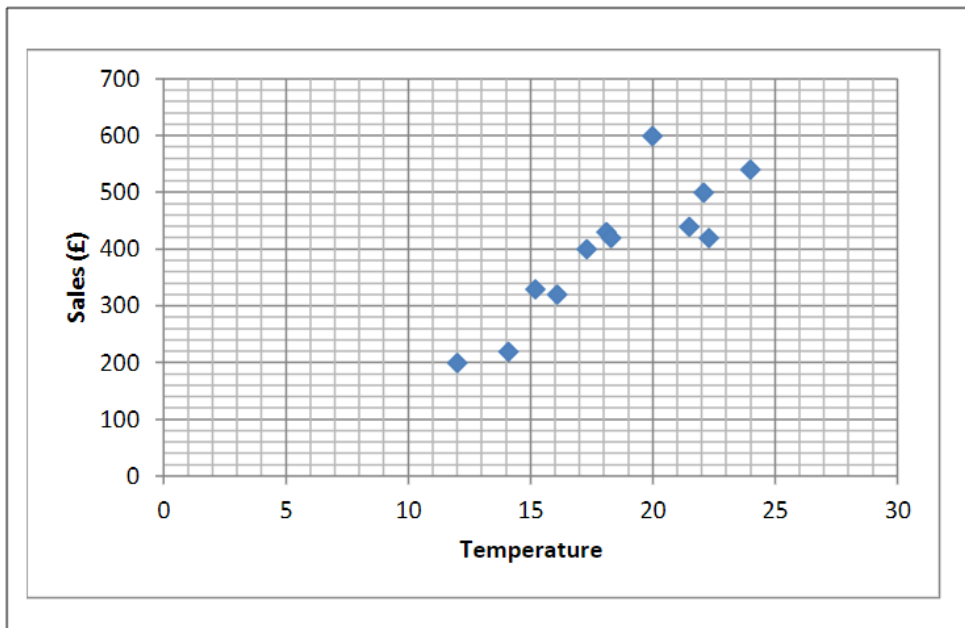
Ranjit says “there is no relationship between the age of a car and its cost” Diagram A

2. Here is a scatter diagram with four points shown. Add 8 points anywhere on the diagram to make a negative correlation



**Answer** – Eight points added. Although the points may be different from those shown below, there should be a general trend from top left to bottom right

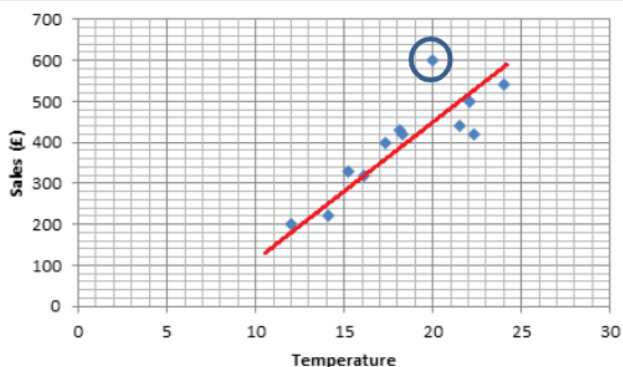
3. An ice cream salesman keeps track of the amount of money taken in ice cream sales against the temperature at midday. The graph below shows this:



- (a) Describe the relationship between the amount of money taken and the temperature at midday.
- (b) Draw in the line of best fit.
- (c) Estimate the amount of money that will be taken if the temperature at midday is 20°C.
- (d) One of the values is an outlier. Give a possible reason for this.

**Answers**

- (a) As the midday temperature increases the amount of money taken increases. (or similar)
- (b) Line drawn on diagram, *similar to the one below*

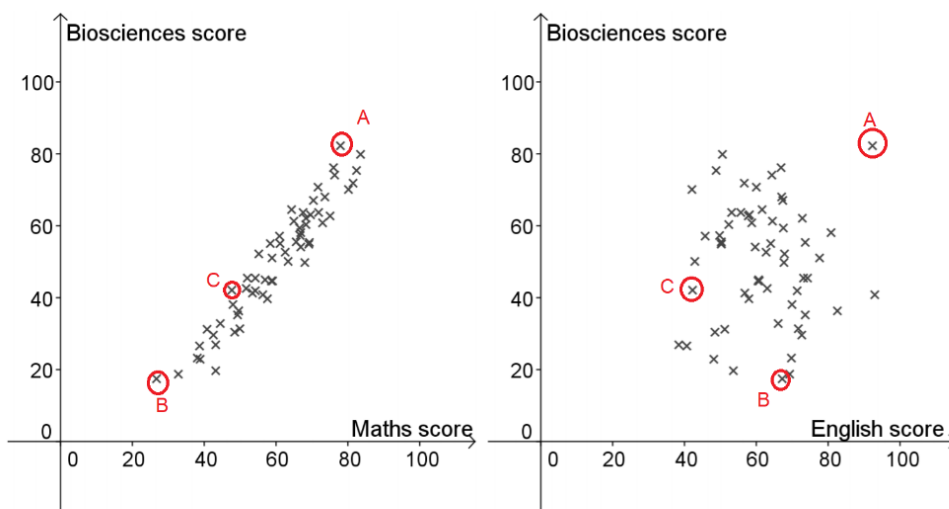


- (c) Read from 20°C in x axis to trendline and across to the y axis, answers around £450.
- (d) The point circled could be an outlier as the takings are higher than what the line would predict them to be. This may be because the day was a bank holiday, or there was a large event on that day.

4. A biosciences lecturer believes that success in the subject depends on being good at both maths and

English. The lecturer wants to investigate whether test scores might show this. Sixty students sit a test in each of maths, English and biosciences. The results are shown in the two scatter diagrams.

**Note** - These data are not real. The scenario is designed to help you think about scatter diagrams.



a,b,c, - shown above

- 5) For each statement, decide how true it is and give a reason.
- (a) 5% of the students scored 20 or less on the biosciences test.
  - (b) One student scored over 80 on all three tests.
  - (c) There is strong positive correlation between maths scores and biosciences scores.
  - (d) If you know that Gemma scored lower than Hui on maths, then it follows that Gemma scored lower than Hui on biosciences.
  - (e) There is a closer correlation between maths and biosciences scores than between English and biosciences scores.
  - (f) Maths scores are a better predictor of success at biosciences than English scores.
  - (g) Success at maths is a bigger influence on success in biosciences than English.
  - (h) The lecturer should give more weight to maths grades than English grades when choosing students for the biosciences course.

#### 5. Answers

- a. 3 out of 60 = 5%, so true
- b. True (Ahmed)
- c. True – the points are close to a straight line with positive gradient
- d. Not necessarily – easy to find a pair of points where one scored higher than other on biosciences but lower on maths.
- e. True. There is no obvious correlation between English and Biosciences scores.
- f. True. If you knew the maths score for a student that would give you a sense of where the Biosciences score will lie.
- g. It is true that maths has a stronger association with biosciences than English does, but 'influence' suggests a causal relationship – this cannot be established from scatter diagrams.
- h. This is a matter for debate. The diagrams show a better association between maths and biosciences than English and biosciences. However, maths may not be a good predictor of success on a future biosciences course. Nevertheless, it may be a sensible approach to consider.

## 4. Standard form

### Standard index form - Using a calculator

You may to watch the clip - using a calculator for standard form calculations – before starting this sheet

Use your calculator to work out the answers to the questions below

1.  $(5.2 \times 10^4) \div (2.5 \times 10^3) = 20.8$
2.  $(5.2 \times 10^4) \times (2.5 \times 10^3) = 130,000,000$  or  $1.3 \times 10^8$
3.  $(3.8 \times 10^{-4}) \div (2.6 \times 10^{-2}) = 0.0146$  (4dp)
4.  $(8.4 \times 10^{-4}) \times (7.6 \times 10^{-3}) = 6.384 \times 10^{-6}$  or 0.000006384
5. On average an adult has 20,000,000,000,000 red blood cells. The mass of a typical red blood cell is 0.0000000001g.
  - a. Write both numbers in standard form  $2 \times 10^{13}$  and  $1 \times 10^{-10}$
  - b. Work out the mass of red blood cells in an average adult  
 $2 \times 10^{13} \times 1 \times 10^{-10} = 2 \times 10^3$  g or 2000g
6. Worldwide, in 2018, there were about 37.8 million flight departures.
  - a. Write this number in standard form  $3.78 \times 10^7$26.3% of these flight departures were European.
  - b. Work out the number of European flights in 2018  
 $3.78 \times 10^7 \times 0.263 = 9941400$

Source <https://www.icao.int/annual-report-2018/Pages/the-world-of-air-transport-in-2018.aspx>

7. The approximate radius of the Earth according to google is 6371 km.

Assume the Earth is approximately a sphere.

- a. Use the formula to surface area of a sphere =  $4\pi r^2$  (where r stands for the radius), to work out the surface area of the Earth in  $\text{km}^2$ .

$$510064471.9 \text{ km}^2$$

- b. A website quotes the surface area of the area of the Earth as  $5.101 \times 10^7 \text{ km}^2$  is this correct?  
NO, it should be  $5.101 \times 10^8 \text{ km}^2$

## 5. Percentages

### Percentages, fractions, and decimals

#### Answers

#### Practice sheet 1

1.

The fraction of the shape that is shaded is  $\frac{3}{8}$

The percentage of the shape that is shaded is  $37.5\%$

2. Complete the statements below

(a) 6 out of 10 =  $60\%$

(b) 1 out of 5 =  $20\%$

3.  $35\%$ ,  $0.56$ ,  $\frac{5}{8}$ ,  $\frac{67}{100}$ ,

4.  $\frac{16}{25} = 0.64 = 64\%$

$\frac{17}{25} = 0.68 = 68\%$

$\frac{6}{25} = 0.24 = 24\%$

$\frac{19}{25} = 0.76 = 76\%$

$\frac{9}{25} = 0.36 = 36\%$

$\frac{11}{25} = 0.44 = 44\%$

$\frac{10}{25} = 0.4 = 40\%$

$\frac{15}{25} = 0.6 = 60\%$

5.

Juice	Amount	Percentage of the drink
Grapefruit	$\frac{3}{8}$ litre	$37.5\%$
Mango	$\frac{1}{8}$ litre	$12.5\%$
Orange	$\frac{1}{2}$ litre	$50\%$

6. (a)  $\frac{170}{205} = 0.829... = 83\%$

(b) Percentage of Alisa's mixture that is water is  $\frac{80}{100} = 80\%$  so Alisa's mixture is **stronger**



7. (a)

(i) Julia scored  $\frac{58}{65} = 89.2\%$  Abdullah scored  $\frac{78}{85} = 91.7\%$  so Abdullah's score is best.

(ii) The tests may not be of equal difficulty. (Other reasons are possible)

(b) Module 1  $\left(\frac{73}{100}\right) = 73\%$  Module 2  $\left(\frac{23}{35}\right) = 65.7\%$  Module 3  $\left(\frac{40}{60}\right) = 66.7\%$  so module 1 is Jason's best performance

8. (a) What fraction of the shape is blue?  $20/36 = 10/18 = 5/9$

(b) What percentage of the shape is blue?  $55.6\%$  (to 1 d.p.)

9. (a) As a fraction the probability of the spinner landing on a shaded section is  $\frac{2}{5}$ .

(b) As a decimal the probability of the spinner landing on a shaded section is 0.4

(c) As a percentage the probability of the spinner landing on a shaded section is  $40\%$

## Percentages, fractions, and decimals

### Answers

#### Practice sheet 2

1. (a) Supermarket A  $72/120 = 60\%$  shaded on the first bar. Supermarket B  $30/40 = 75\%$  shaded on the second bar.

(b) 32 out of 80 and 77 out of 110.

2.  $1/5$  shaded  $1/5 = 20\%$

3.  $2/7 = 0.285\dots$  so  $29\%$  to the nearest  $1\%$

4. Supermarket C ( $40/60 = 66.7\%$ ) Supermarket A ( $65/110 = 59.1\%$ ) Supermarket B ( $52/100 = 52\%$ )

5. (a)  $103/645 = 15.9... = 16\%$

(b)  $£195.70/£1457.75 = 13.4... = 13\%$

6. (a) Public services

(b)  $12/200 = 6\%$

(c)  $28/50 = 56\%$

## Fractions, decimals and percentages

### Practice sheet 3 Answers

1.  $£6 - £4.50 = £1.50$

$£1.50/£4.50 = 0.333... = 33.3\%$

2. (a)  $4/12 = 0.333... = 33.3\%$

(b) **Company B's** increase is  $48/92 = 52.2\%$  which is greater than Company A.

3. (a)  $£25028.60 - £2291.00 = 22737.60$

$22737.60/2291 = 9.924... \text{ so } 992\%$

(b)  $198000 - 11000 = 187000$

$187000/11000 = 17.0 \text{ so } 1700\%$

4. With 50 children joining the club now has 250 members.

Girls now account for 20% of 250 = 50 girls

There were 30 girls before and now there 50 which means 20 of the new 50 children were girls.

So 30 of the new children must have been boys.

This makes 70 boys now at the club.

5. (a)  $£1.45 - £1.07 = £0.38$  or 38p

(b)  $58/145 = 0.4 = 40\%$

(c) 3p

(d)  $3/61 = 0.491.. = 4.9\%$

## Reverse percentages

### Practice sheet 1 Answers

1.  $25\% = 100$

$1\% = 4$

$100\% = 400$

The number is 400

2.  $35\% = 1050$

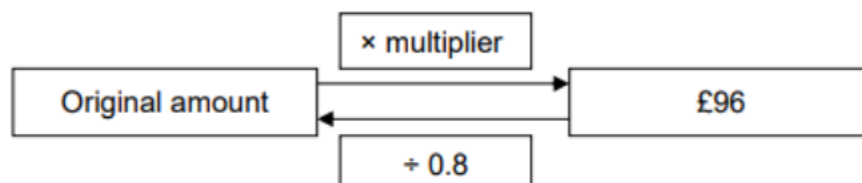
$1\% = 30$

$65\% = 1950$

The number is 1950

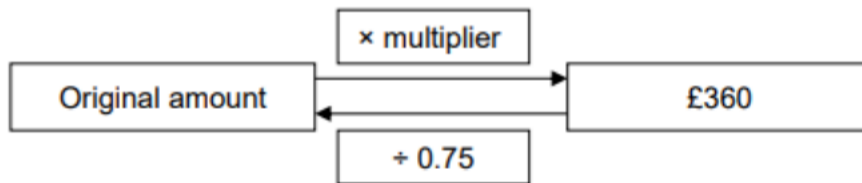
3.  $£96 \div 0.8 = £120$

The original price of the jacket before the reduction was £120



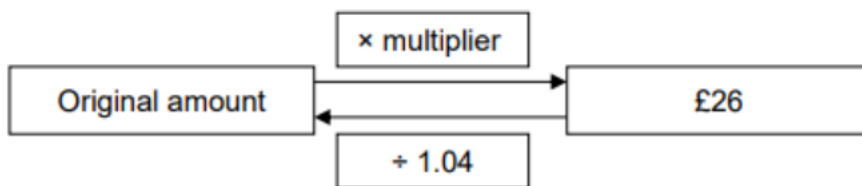
$$4. \quad £360 \div 0.75 = £480$$

The original price of the bed before the reduction was £480



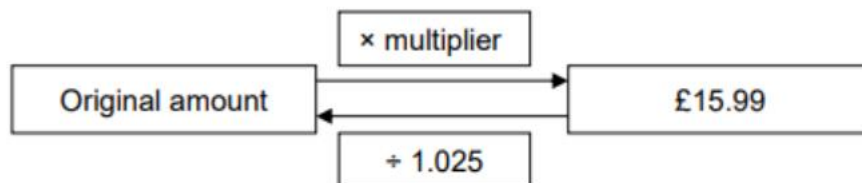
$$5. \quad £26 \div 1.04 = £25$$

The original price of the train fare before the increase was £25



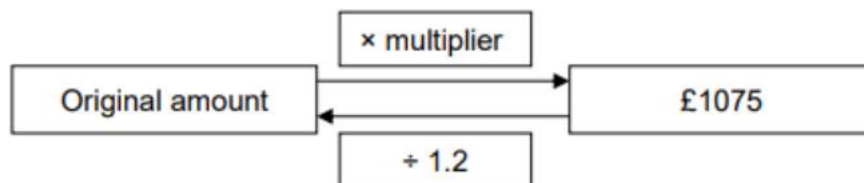
$$6. \quad £15.99 \div 1.025 = £15.60$$

The original price of steak and chips before the increase was £15.60



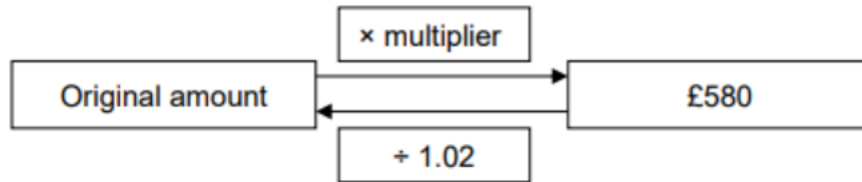
$$7. \quad £1075 \div 1.2 = £895.83$$

The price of the holiday without VAT is £895.83



8.  $£580 \div 1.02 = £568.63$

The price of the antique ring sold privately is £568.63

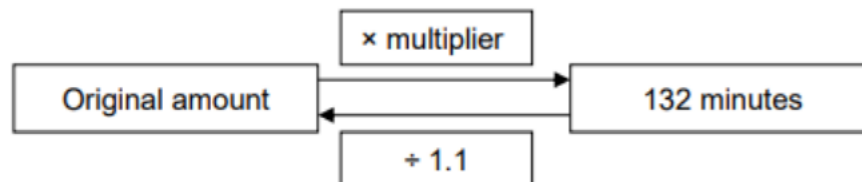


9. 2 hours and 12 minutes = 132 minutes

$132 \text{ minutes} \div 1.1 = 120 \text{ minutes}$

$120 \text{ minutes} = 2 \text{ hours}$

The winner took 2 hours to complete the race



10.  $500 \div 1.4 = 357.1428571$  which would make a fraction of a person therefore the percentage increase cannot be exactly 40%

## Repeated percentage change

### Answers

1.  $£12000 \times 1.033 = £13112.72$

$£13112.72 - £12000 = £1112.72$

Louisa will earn interest of £1112.72

2.  $£5000 \times 1.024 = £5412.16$

The total amount Ed will have after four years will be £5412.16

3.  $£7000 \times 1.0255 = £7919.86$

$£7000 \times 1.0256 = £8117.85$

It will take 6 complete years for the investment to exceed £8000.

4.  $£2300 \times 0.9653 = £2066.85$

After three years the watch will be worth £2066.85

5.  $£9000 \times 0.9254 = £6588.85$

After four years the bike will be worth £6600 (to the nearest £100)

6. (a)  $600 \div 15000 \times 100 = 4\%$

The percentage decrease in the value of the painting was 4%

(b)  $£14400 \times 0.967 = £10820.84$

The painting will be worth £10820.84 after a further 7 years

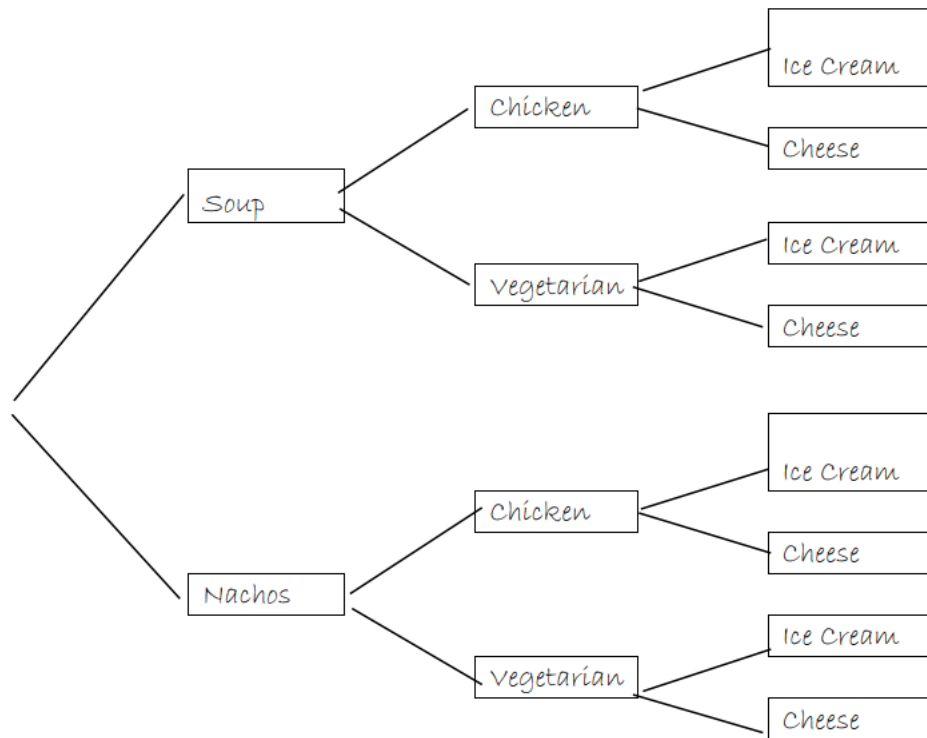
7.  $£230000 \times 1.0454 = £274279.28$

After 4 years the house will be worth £274000 (to the nearest £1000)

## 6. Probability

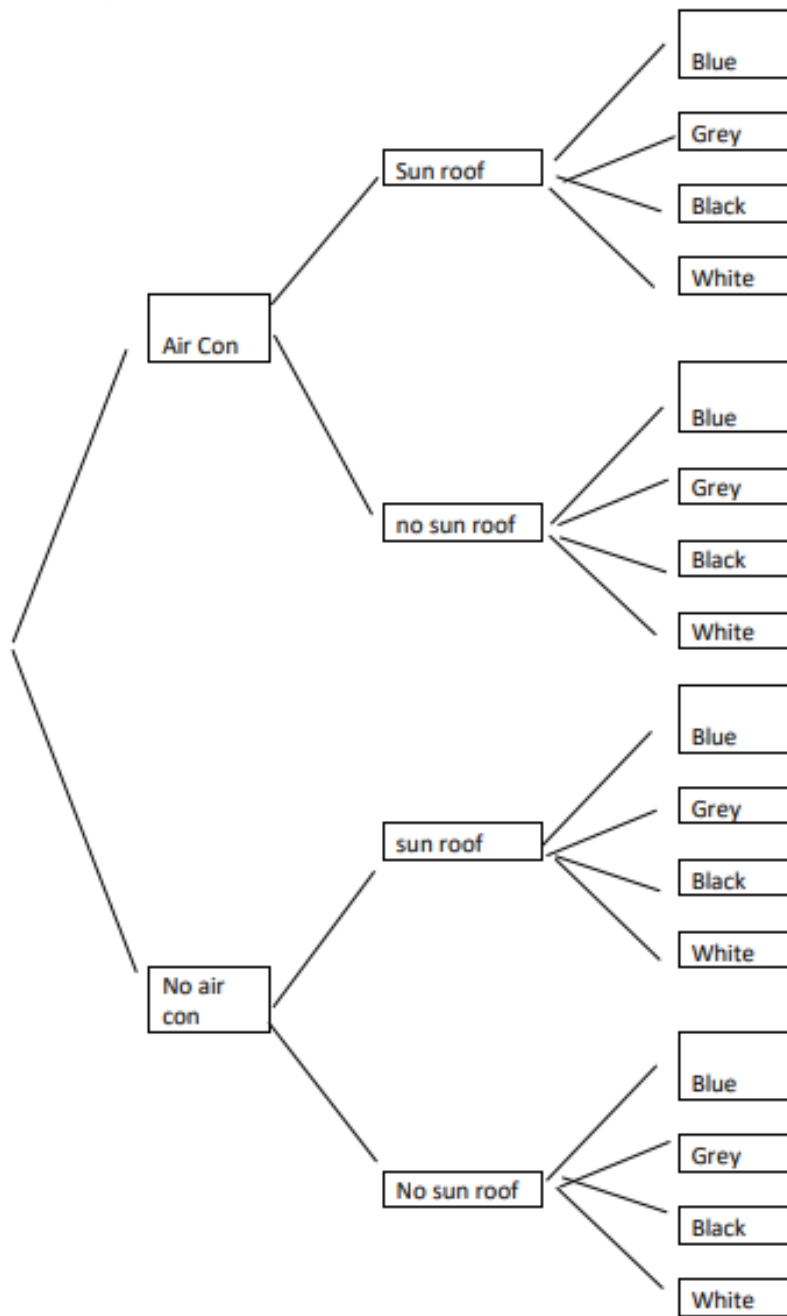
### Solutions: Outcomes

1. a)



b) There are 8 equally likely outcomes. 2 of these contain both soup and the vegetarian option. So the probability is  $\frac{2}{8} = \frac{1}{4}$

2. a)



b) There are 16 equally likely outcomes altogether. 4 of these outcomes have sunroof and grey or sunroof and white. So the probability is  $\frac{4}{16} = \frac{1}{4}$



3. number of one digit binary numbers = 2 (0 and 1)  
 number of two digit binary numbers = 2 (10 and 11)

number of three digit binary numbers = 4 (100, 101, 110, 111).

There is only one possibility for first digit, then two possibilities for the second digit, then two possibilities for the third digit.  $1 \times 2 \times 2 = 4$

Four digits:

There is one possibility for the first digit, two for the second digit, two for the third digit and two for the fourth digit.

Number of four digit numbers =  $1 \times 2 \times 2 \times 2 = 8$

Six digits:

There is one possibility for the first digit, two for the second digit, two for the third digit, two for the fourth digit, two for the fifth digit and two for the sixth digit.

Number of four digit numbers =  $1 \times 2 \times 2 \times 2 \times 2 \times 2 = 32$ .

## Solutions: Working out relative frequencies

1. Tom has won 16 times out of 30. An estimate of the probability he will win the next game is  $\frac{16}{30} = \frac{8}{15}$
  
2. It has rained on St Swithin's Day on 6 out of the last 20 times. So an estimate of the probability is  $\frac{6}{20} = \frac{3}{10} = 30\%$
  
3. Chris's team has lost 21 out of the last 44 matches. An estimate of the probability the team will lose the next one is  $\frac{21}{44}$
  
4. The weather forecaster has been correct for 15 out of 30 days. An estimate of the probability he will be correct the next day is 50%
  
5. Kate has taken more than 30 minutes to travel to work on 4 out of 20 days. The probability she will take more than 30 minutes on the next day is  $\frac{4}{20} = \frac{1}{5} = 20\%$

## Solutions: Two-way tables

### Question 1

a) There are 151 women MPs and 650 MPs altogether

So the probability a woman is chosen is  $= \frac{151}{650}$

b) There are 306 Conservative MPs and 650 MPs altogether.

So the probability a Conservative MP is chosen is  $= \frac{306}{650}$

c) There are 7 female Liberal Democrats and 650 MPs altogether

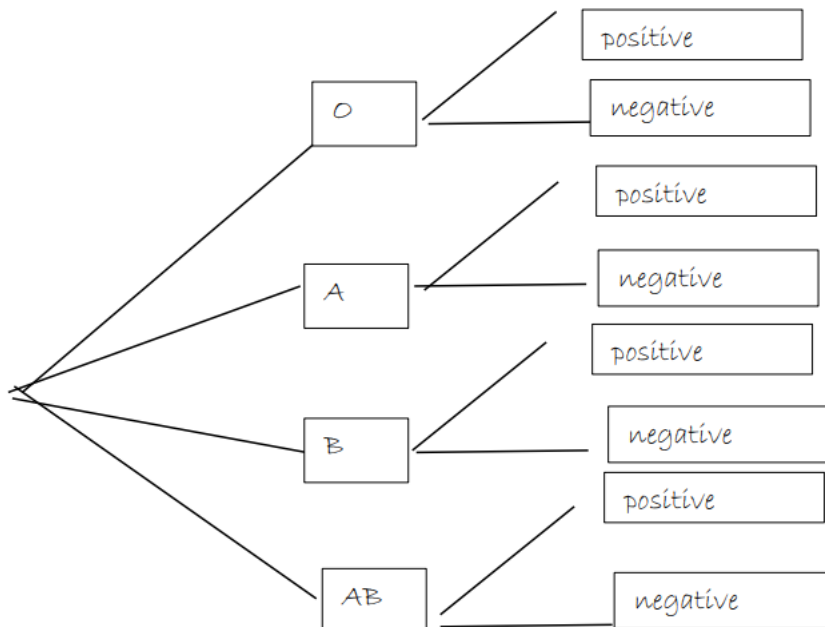
So the probability a female Liberal Democrat MP is chosen is  $= \frac{7}{650}$

d) There are  $650 - 306 = 344$  MPs who are not Conservative and 650 MPs altogether

So the probability someone who is not a Conservative is chosen is  $= \frac{344}{650} = \frac{172}{325}$

### Question 2

a



b) The percentage of people with B negative blood = 2%. This would be the probability

c) The percentage of people with blood A is 42%. The percentage with positive blood is 7%

$$\text{The probability is } \frac{7}{42} = \frac{1}{6}$$

d) The percentage of people with positive blood is 83% - read this from the table.

### Question 3

a) The number of people passing both tests first time = 24

$$\text{So an estimate of the probability} = \frac{24}{100} = \frac{6}{25}$$

a) The number of people who passed one test and failed one = 21 + 32 = 53

$$\text{So an estimate of the probability} = \frac{53}{100}$$

b) The number of people who failed the theory test first time = 44

The number of people who passed the practical but failed the theory = 21

$$\text{So an estimate of the probability} = \frac{21}{44}$$

### Question 4

a) There are 240 people altogether.

There are 100 + 80 people in the control group or given the placebo

$$\text{So the probability} = \frac{180}{240} = \frac{3}{4}$$

b) There were 60 people given the drug, 47 of these 60 people improved.

$$\text{So an estimate of the probability} = \frac{47}{60}$$

c) There were 144 people who improved.

Of these 47 had taken the drug

$$\text{So an estimate of the probability} = \frac{47}{144}$$

## 7. Proportional reasoning

### Proportional reasoning

#### Practice sheet Answers

- 40 kph + 40 kilometres per hour
  - $3 \times 40 = 120$  km
  - $2.5 \times 40 = 100$  km
  - $60 \div 15 = 4$   
15 minutes is a quarter of an hour  
 $40 \div 4 = 10$  km
  - $60 \div 12 = 5$   
12 minutes is one fifth of an hour  
 $40 \div 5 = 8$  km
- $\text{£}1 = \text{\$}1.32$     $\text{£}1 = \text{€}1.06$ 
  - $\text{£}20 = 20 \times 1.32 = \text{\$}26.40$
  - $\text{£}36 = 36 \times 1.32 = \text{\$}47.52$
  - $\text{£}78 = 78 \times 1.06 = \text{€}82.68$
  - $\text{\$}15.84 = 15.84 \div 1.32 = \text{£}12$
  - $\text{€}44.52 = 44.52 \div 1.06 = \text{£}42$
- $\text{£}1 = \text{₹}85$ 
  - $\text{₹}8250 = 8250 \div 85 = \text{£}97.06$
  - Fazal
- Conversion rate at the time  $\text{£}1 = \text{€}1.10$ 
  - Republic of Ireland  $\text{€}1.32$  per litre of petrol  
UK  $\text{£}1.22$  per litre of petrol =  $1.22 \times 1.10 = \text{€}1.34$   
OR  
UK  $\text{£}1.22$  per litre of petrol  
Republic of Ireland  $\text{€}1.32$  per litre of petrol =  $1.32 \div 1.10 = \text{£}1.20$   
Republic of Ireland has the cheapest petrol per litre
  - $\text{€}0.02$  OR  $\text{£}0.02$
- 5 text books cost  $\text{£}29$   
1 text book cost  $\text{£}5.80$   
9 text books cost  $\text{£}52.20$
- 30 pens cost  $\text{£}4.50$ 
  - 30 pens cost  $\text{£}4.50$   
1 pen costs  $\text{£}0.15$   
40 pens cost  $\text{£}6$
  - $\text{£}0.15$  for 1 pen  
 $100 \div 0.15 = 666.66\dots$  pens  
 $\text{£}100$  would buy 666 pens

7. 40 litres of fuel to travel 140 km

(a)

Distance	Fuel needed
140 km	40 litres
1 km	$40 \div 140 = 0.2857\dots$
98 km	$0.2857\dots \times 98 = 28$ litres (to nearest litre)

(b)

Distance	Fuel needed
140 km	40 litres
$140 \div 40 = 3.5$ km	1 litre
$75 \times 3.5 = 262.5$ km	75 litres

8. 5 snack bars which cost £2.55, 8 snack bars which cost £3.92

(a) 5 bars cost £2.55

1 bar costs £0.51

(b) 8 bars cost £3.92

1 bar costs £0.49

(c) The pack of 8 bars is best value for money

9. 12 pages per minute

1 page per 5 seconds

100 pages per 500 seconds = 8 minutes and 20 seconds

10. History she scored 56 out of 80 - 70%

Geography she scored 73 out of 100 - 73%

73 out of 100 is a higher percentage

11. 5 miles = 8 km

1 km = 0.625 mile

56kph =  $56 \times 0.625 = 35$ mph

12. 5 miles = 8 km

(a) 400 miles =  $80 \times 8 = 640$  km

(b) 30 litres for 600 km

1 litre for 20 km

8 litres for 160 km

## 8. Algebra

### Practice Questions: Solving equations

1) Solve the following equations.

(a)  $2x + 3 = 9$

(b)  $3w - 1 = 14$

(c)  $7y + 2 = 30$

(d)  $5x + 20 = 35$

(e)  $6c - 12 = 48$

(f)  $8m - 4 = 20$

(g)  $7w + 13 = 90$

(h)  $12p - 18 = 30$

(i)  $9w - 5 = 67$

2) Solve the following equations.

(a)  $4x + 1 = 2x + 7$

(b)  $5x + 4 = 3x + 16$

(c)  $2x + 8 = x + 12$

(d)  $7x + 1 = 2x + 46$

(e)  $6x - 3 = 2x + 13$

(f)  $9x - 10 = 7x + 24$

3) Solve the following equations.

(a)  $5(x + 3) = 3(x + 9)$

(b)  $8(x - 1) = 4(x + 3)$

(c)  $3(x + 13) = 10(x - 1)$

Question 1:

(a)  $x = 3$

(b)  $w = 5$

(c)  $y = 4$

(d)  $x = 3$

(e)  $c = 10$

(f)  $m = 3$

(g)  $w = 11$

(h)  $p = 4$

(i)  $I = 8$

Question 2:

(a)  $x = 3$

(b)  $x = 6$

(c)  $x = 4$

(d)  $x = 9$

(e)  $x = 4$

(f)  $x = 17$

Question 3:

(a)  $x = 6$

(b)  $x = 5$

(c)  $x = 7$

### Practice Questions: Solving inequalities

1) Solve each of the inequalities:

(a)  $x + 4 > 9$     (b)  $x - 3 < 2$     (c)  $2x \leq 14$     (d)  $8x < 16$

2) Solve each of the inequalities:

(a)  $2x + 1 \leq 9$     (b)  $3x - 5 > 16$     (c)  $4x + 8 < 32$     (d)  $5x - 2 \geq 68$

3) Solve each of the inequalities:

(a)  $4x + 3 > 2x + 11$                       (b)  $x + 1 \geq 3x - 18$   
(c)  $13x - 12 < 3x + 13$                       (d)  $7x - 5 \geq 3x + 11$

Question 1:

(a)  $x > 5$                       (b)  $x < 5$                       (c)  $x \leq 7$                       (d)  $x < 2$

Question 2:

(a)  $x \leq 4$                       (b)  $x > 7$                       (c)  $x < 6$                       (d)  $x \geq 14$

Question 3:

(a)  $x > 4$                       (b)  $9.5 \geq x$                       (c)  $x < 2.5$                       (d)  $x \geq 4$

### Practice Questions: Solving simultaneous equations

1) Solve the following by elimination:

(a)  $6x + y = 18$   
 $4x + y = 14$

(b)  $4x + 2y = 10$   
 $x + 2y = 7$

(c)  $9x - 4y = 19$   
 $4x + 4y = 20$

(d)  $9x - 4y = 59$   
 $2x - y = 12$

(e)  $2x + 8y = 43$   
 $x + 3y = 18$

(f)  $6x + 3y = 45$   
 $2x - 2y = 12$

(g)  $3x + 8y = 31$   
 $5x + 3y = 31$

(h)  $7x - 15y = 2.5$   
 $3x - 2y = 5.5$

(i)  $3x + 2y = 53$   
 $2x + 5y = 72$

Answers:

(a)  $x=2$   
 $y=6$

(b)  $x=1$   
 $y=3$

(c)  $x=3$   
 $y=2$

(d)  $x=11$   
 $y=10$

(e)  $x=7.5$   
 $y=3.5$

(f)  $x=7$   
 $y=1$

(g)  $x=5$   
 $y=2$

(h)  $x=2.5$   
 $y=1$

(i)  $x=11$   
 $y=10$